Parallel Neuro-Tabu Search Algorithm for the Job Shop Scheduling Problem

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Abstract. We propose two parallel algorithms based on neuro-tabu search method, designed to solve the job shop problem of scheduling. The first algorithm is based on independent runs of the neuro-tabu with different starting points. The second one uses sophisticated diversification method based on path-relinking methodology applied to the set of elite solutions. Proposed approaches are especially effective for the instances of large size.

1 Introduction

We consider the job shop scheduling problem here, which can be described as follows (see [4]). There is a set of jobs and a set of machines. Each job consists of a number of operations which have to be processed in a given order, each one on a specified machine during a fixed time. The processing of an operation cannot be interrupted. Each machine can process at most one operation at a time. We want to find a schedule (the assignment of operations to time intervals on machines) that minimizes the makespan. The job shop scheduling problem, although relatively easily stated, is strongly NP-hard and it is considered as one of the hardest problems in the area of combinatorial optimization.

Because of NP-hardness of the problem heuristics and metaheuristics are recommended as ‘the most reasonable’ solution methods. The majority of these methods refer to the makespan minimization. We mention here a few recent studies: Jain, Rangaswamy, and Meeran [21]; Pezzella and Merelli [30]; Grabowski and Wodecki [17]; Nowicki and Smutnicki [27]; Bożejko and Uchoński [6]. Heuristics algorithms based on dispatching rules are also proposed in papers of Holthaus and Rajendran [19], Bushee and Svestka [10] for the problem under consideration. For the other regular criteria such as the total tardiness there are proposed

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metaheuristics based on various local search techniques: simulated annealing
[34], tabu search [2] and genetic search [26].

Here we propose the new approach to the distributed tabu search metaheuristic
designing to solve difficult discrete optimization problems, such as the job
shop problem, using a multi-GPU cluster with distributed memory. We also
determine theoretical number of processors, for which the speedup measure has a
maximum value. We experimentally determine what parallel execution time $T_p$
can be obtained in real-world installations of multi-GPU clusters (nVidia Tesla
S2050 with a 6-cores CPU server) for Taillard benchmarks [32] of the job shop
scheduling problem, and compare them with theoretically determined values.

2 Job Shop Problem

Job shop scheduling problems result from many real-world cases, which means
that they have good practical applications as well as industrial significance. Let
us consider a set of jobs $J = \{1, 2, \ldots, n\}$, a set of machines $M = \{1, 2, \ldots, m\}$
and a set of operations $O = \{1, 2, \ldots, o\}$. The set $O$ is decomposed into subsets
connected with jobs. A job $j$ consists of a sequence of $o_j$ operations indexed
consecutively by $(l_{j-1}+1, l_{j-1}+2, \ldots, l_j)$, which have to be executed in this
order, where $l_j = \sum_{i=1}^j o_i$ is the total number of operations of the first $j$ jobs.
$j = 1, 2, \ldots, n$. $l_0 = 0$. $\sum_{i=1}^n o_i = o$. An operation $i$ has to be executed on
machine $c_i \in M$ without any idleness in time $p_i > 0$. $i \in O$. Each machine can
execute at most one operation at a time. A feasible solution constitutes a vector
of times of the operation execution beginning $S = (S_1, S_2, \ldots, S_o)$ such that the
following constraints are fulfilled

$$S_{l_i-1+1} \geq 0, \quad j = 1, 2, \ldots, n. \quad (1)$$

$$S_i + p_i \leq S_{i+1}, \quad i = l_{j-1}+1, \quad l_{j-1}+2, \ldots, l_j-1. \quad j = 1, 2, \ldots, n. \quad (2)$$

$$S_i + p_i \leq S_j \quad \text{or} \quad S_j + p_j \leq S_i, \quad i, j \in O. \quad v_i = v_j, \quad i \neq j. \quad (3)$$

Certainly, $C_j = S_j + p_j$. An appropriate criterion function has to be added to the
above constraints. The most frequent are the following two criteria: minimization
of the time of finishing all the jobs and minimization of the sum of job finishing
times. From the formulation of the problem we obtain $C_j \equiv C_{l_j}, \quad j \in J$.

The first criterion, the time of finishing all the jobs

$$C_{\max}(S) = \max_{1 \leq j \leq n} C_{l_j}. \quad (4)$$

corresponds to the problem denoted as $J||C_{\max}$ in the literature. The second
criterion, the sum of job finishing times

$$C(S) = \sum_{j=1}^n C_{l_j}. \quad (5)$$

corresponds to the problem denoted as $J||\sum C$, in the literature.
Both problems described are strongly NP-hard and although they are similarly modelled, the second one is found to be harder because of the lack of some specific properties (so-called block properties, see [27]) used in optimization of execution time of solution algorithms.

3 Tabu Search Mechanism with Neural Network Application

In the considered neuro-tabu search algorithm NTS each move is represented by its neuron. For the neighborhood considered in [28] a network of neurons formed of \( o - 1 \) neurons. Let \( i \)-th neuron represents a move consisting in swap of two adjacent elements on the positions \( i \) and \( i + 1 \) in a solution \( \pi \).

![Neural Network Diagram](image)

**Fig. 1.** Dependencies between activation of the neuron and a move

In a proposed neural network architecture a history of each neuron is stored as its internal state (tabu effect). If in an iteration neuron is activated, then the value 1 is fixed on its output and values 0 are fixed on the outputs of other neurons. The neuron activated in an iteration must not be activated once again for the next \( s \) iterations. Each neuron is defined by the following equations:

\[
\eta_i(t + 1) = \alpha \Delta_i(t),
\]

\[
\Delta_i(t) = \frac{C_{max}(\pi_v^{(t)}) - C_{max}^*}{C_{max}^*}.
\]

\[
\gamma_i(t + 1) = \sum_{d=0}^{s-1} k^d x_i(t - d).
\]

where \( x_i(t) \) is an output of the neuron \( i \) in the iteration \( t \). Symbol \( C_{max}(\pi_v^{(t)}) \) means the value of the goal function for the permutation obtained after executing
a move \( v \) in the iteration \( t \), i.e. \( \pi(t) \). Symbol \( \Delta_i(t) \) means a normalized, current value of the goal function, and \( C_{\max}^* \) is the value of the best solution found so far. Parameters \( \alpha \) and \( k \) are scale factors. A symbol \( \eta_i(t + 1) \) (gain effect) defines quality of a move \( v \). A variable \( \gamma_i(t + 1) \) (tabu effect) stores a history of the neuron \( i \) for the last \( s \) iterations. Neuron is activated if it has a low value of the tabu effect and it gives a better reduction of the \( C_{\max} \). More detailed neuron \( i \) is activated if it has the lowest \( \{ \eta_i(t + 1) + \gamma_i(t + 1) \} \) value of all the neurons.

If \( 0 < k < 1 \) \( i \) \( s = \tau \) then an equation (8) takes the form of:

\[
\gamma_i(t + 1) = k\gamma_i(t) + x_i(t).
\]

where \( \gamma_i(0) = 0 \) and \( x_i(0) = 0 \) for each \( i \). From the equation (9) it follows, that the value of \( \gamma_i(t) \) of each neuron decreases exponentially (Fig. 3 and 4).

In many algorithms proposed in the literature which are based on the tabu search method a so-called aspiration criterion is implemented. It consists in executing the forbidden move if it follows to the base solution with the goal function value lower than the best found so far.

In the proposed neuro-tabu search such a function can be implemented by ignoring tabu effect for a move \( v \) for which \( \Delta_i < 0 \). However during computational experiments we have observed that it does not give good effects. Therefore a proposed neuro-tabu search has not got such a function. The skeleton of the neuro-tabu search algorithm is given on the Fig. 2.

4 Parallel Neuro-Tabu Search Algorithm \( pNTS \)

Here we propose a solution method to the job shop problem in the distributed computing environments, such as multi-GPU clusters. Tabu search algorithm is executed in concurrent working threads, as in multiple-walk model of parallelization [1] (MPDS. Multiple starting Point Different Strategies in the Voß [33] classification of parallel tabu search metaheuristic). Additionally, MPI library is used to distribute calculations among GPUs (see Fig. 4).

Now let us consider a single cycle of the MPI data broadcasting, multi-GPU computations and batching up of the results obtained. Let us assume that the single communication procedure between two nodes of a cluster takes the time \( T_{\text{comm}} \), the time of sequential tabu search computations is \( T_{\text{seq}} \) and the computations time of parallel tabu search is \( T_{\text{calc}} = \frac{T_{\text{seq}}}{p} \) (\( p \) is the number of GPUs). Therefore, the total parallel computations time of the single cycle is

\[
T_p = 2T_{\text{comm}} \log_2 p + T_{\text{calc}} = 2T_{\text{comm}} \log_2 p + \frac{T_{\text{seq}}}{p}.
\]

In case of using more processors, the parallel computing time \( \frac{T_{\text{calc}}}{p} \) decreases, whereas the time of communication \( 2T_{\text{comm}} \log p \) increases. We are looking for such a number of processors \( p \) (let us call it \( p^* \)) for which \( T_p \) is minimal. By calculating \( \frac{\partial T_p}{\partial p} = 0 \) we obtain

\[
\frac{2T_{\text{comm}}}{p \ln 2} - \frac{T_{\text{seq}}}{p^2} = 0
\]
Fig. 2. A skeleton of the neuro-tabu search algorithm

and then

$$p = p^* = \frac{T_{seq} \ln 2}{2T_{comm}},$$

(11)

which provides us with an optimal number of processors $p^*$ which minimizes the value of the parallel running time $T_p$.

The fraction of communication time is $O(\log_2 p)$ in this tree-based data broadcasting method, therefore this is another situation than for linear-time broadcasting (discussed in [3]), for which the overall communication and calculation
efficiency is much lower. On the other hand, the linear-time broadcasting is similar to described by Brooks’ Law [9] for project management, i.e., the expected advantage from splitting development work among \( n \) programmers is \( O(n) \) but the communications cost associated with coordinating and then merging their work is \( O(n^2) \).

5 Advanced Neuro-Tabu Search Algorithm \( iNTS \)

As the second solution method we propose an approach introduced by Nowicki and Smutnicki [27] with using neuro-tabu instead of classic tabu search algorithm, as in the original paper. The proposed \( iNTS \) algorithm operates on the set of dispersed (elite) solution obtained with using \( NIS(\gamma, \delta, C^R) \) function (see Fig. 6), where \( \gamma, \delta \) are two processing orders and \( C^R \) is the reference makespan (goal function value). Inside, the neighborhood generation function is used based
on swap moves of adjacent operations (see Bożejko [4]). We denote by $N(\pi)$ the set of moves obtained from a solution $\pi$ by swapping any two adjacent operations lying on critical path (to obtain feasible solution, see [17]). The fundamental aim of the $NIS$ function is to provide the solution $\varphi = NIS(\gamma, \delta, C^{R})$ located between $\gamma$ and $\delta$ reference solutions to run a neuro-tabu search explanation in the main $INTS$ algorithm (see Fig. 7). To control the distance between any two solution $\alpha, \beta$, the Kendall’s tau measure $D(\alpha, \beta)$ is used to represent the minimal number of adjacent swap moves (inversions) to obtain permutation $\beta^{-1}$ from $\alpha^{-1}$ (for more measures on permutations see e.g. Diaconis [13]). Values of tuning parameters were following: $maxE = 8$ (number of elite solutions), $maxD = 5$ (maximal distance used in the loop), $maxV = 0.5$.

Algorithm 1. $NIS(\gamma, \delta, C^{R})$
Input: $\gamma, \delta$ - two processing orders; $C^{R}$ - reference makespan;
Output: $\varphi$ - processing order; update reference makespan $C^{R}$;
$\pi \leftarrow \gamma$; $\text{iter} \leftarrow 0$. Find $\delta^{-1}$ and $D(\gamma, \delta)$

repeat
  $\text{iter} \leftarrow \text{iter} + 1$; Find $N(\pi)$;
  For any $\upsilon \in N(\pi)$ calculate and store $C_{\text{max}}(\pi(\upsilon))$;
  Find $N^{+} = \{\upsilon = (x, y) \in N(\pi) : \delta^{-1}(y) < \delta^{-1}(x)\}$;
  if $N^{+} \neq \emptyset$ then $K \leftarrow N^{+}$ else $K \leftarrow N(\pi)$;
  Select the move $\upsilon \in K$ such, that
    $C_{\text{max}}(\pi(\upsilon)) = \min_{\upsilon \in K} C_{\text{max}}(\pi(\upsilon))$;
  Denote $\pi(\upsilon)$ by $\alpha$;
  $\pi \leftarrow \alpha$; $\varphi \leftarrow \pi$;
  if $C_{\text{max}}(\pi) < C^{R}$ then $C^{R} \leftarrow C_{\text{max}}(\pi)$ and exit;
until $\text{iter} \geq maxV \cdot D(\gamma, \delta)$; \{maxV $\in (0, 1)$ - parameter\}

Fig. 6. $NIS(\gamma, \delta, C^{R})$ function

6 Computational Experiments

Proposed algorithms were ran on the server based on 6-cores Intel Core i7 CPU X980 (3.33GHz) processor equipped with nVidia Tesla S2050 GPU (1792 cores) working under 64-bit Linux Ubuntu 10.04.4 LTS operating system and tested on the benchmark problems taken from Taillard [32].
Algorithm 2. \textit{iNTS}

\textbf{Input:} $\pi^0$ – processing orders provided by INSA:
\textbf{Output:} $\pi^*$ – the best found processing order
\text{and its makespan $C^*$}:

Set $(\pi^1, C^1) \leftarrow \text{NTS}(\pi^0)$ and $C^* \leftarrow C^1$;

For $i \leftarrow 2, \ldots, \text{maxE}$ do

\begin{itemize}
  \item $\varphi \leftarrow \text{NIS}(\pi^{i-1}, \pi^0, C^*)$;
  \item $(\pi^i, C^i) \leftarrow \text{NTS}(\varphi)$;
  \item $C^* = \min\{C^*, C^i\}$;
\end{itemize}

Repeat

Find $1 \leq l \leq \text{maxE}$ so that

$D(\pi^k, \pi^l) = \max\{D(\pi^k, \pi^i) : 1 \leq i \leq \text{maxE}\};$

Set $\varphi = \text{NIS}(\pi^k, \pi^l, C^*$) and $(\pi^l, C^l) \leftarrow \text{NTS}(\varphi)$;

If $C^l < C^k$ then set $(\pi^*, C^*) \leftarrow (\pi^l, C^l)$ and $k \leftarrow l$;\n
Until $\max\{D(\pi^k, \pi^i) : 1 \leq i \leq \text{maxE}\} < \text{maxD}$.

\textbf{Fig. 7. Algorithm \textit{iNTS}}

Computational experiments results are compared in the Table 1. Particular columns have the following notion:

\begin{itemize}
  \item \textit{sNTS} – sequential Neuro Tabu Search algorithm of Bożejko and Uchoński [6].
  \item \textit{pNTS} – parallel (for $p = 16$) Neuro Tabu Search algorithm. MPSS model
    (due to the Voß [33] classification) without communication (proposed in the
    Section 4); for each processor the starting solution was generated by the
    \textit{NTS} algorithm executed for $\text{process.id} \times 100$ iterations.
  \item \textit{iNTS} – advanced \textit{NTS} algorithm based on the diversification and intensification
    methodology proposed in the Section 5.
\end{itemize}

Processing times for all benchmark instances were: (sequential) \textit{NTS} – 132m27.319s, (parallel) \textit{pNTS} ($p = 4$) – 169m45.748s (longer time of parallel algorithm work causes of the method of starting solutions generation and synchronization). \textit{iNTS} – about 60 hours. As we can observe the proposed \textit{iNTS} algorithm managed to obtain the average relative percentage deviation from the

\begin{table}[h]
\centering
\small
\begin{tabular}{|c|c|c|c|c|}
\hline
problem & \multicolumn{1}{c|}{n \times m} & \multicolumn{1}{c|}{sNTS} & \multicolumn{1}{c|}{pNTS ($p = 16$)} & \multicolumn{1}{c|}{iNTS} \\
\hline
TA01-10 & 15 \times 15 & 0.4948 & 0.3141 & 0.0652 \\
TA11-20 & 20 \times 15 & 1.1691 & 0.9129 & 0.4412 \\
TA21-30 & 20 \times 20 & 1.2486 & 0.7033 & 0.4803 \\
TA31-40 & 30 \times 15 & 1.0592 & 0.7965 & 0.3764 \\
TA41-50 & 30 \times 20 & 1.8565 & 1.5634 & 0.8328 \\
TA51-60 & 50 \times 15 & 0.0915 & 0.0915 & 0.0520 \\
TA61-70 & 50 \times 20 & 0.1479 & 0.0210 & 0.0140 \\
TA71-80 & 100 \times 20 & 0.0090 & 0.0090 & 0.0090 \\
\hline
average & & 0.7596 & 0.5515 & 0.2839 \\
\hline
\end{tabular}
\caption{Percentage relative deviations (PRD) to the best known solutions}
\end{table}
best known solution of the Taillard instances on the level of 0.28%. Average PRD values for test instances of the group TA61-TA70 (1,000 operations) are 10 times lower than for the proposed iNTS algorithm. Comparing results for sNTS and pNTS algorithm we can notice that using parallel algorithm results in obtaining much lower PRD values. Based on this we plan to implement parallel version of iNTS algorithm as a future work.

7 Conclusions

In this paper we propose an approach designed to solve difficult problems of combinatorial optimization in distributed parallel architectures without shared memory, such as clusters of nodes equipped with GPU units (i.e. multi-GPU clusters). The methodology can be especially effective for large instances of hard to solve optimization problems, such as flexible scheduling problems as well as discrete routing and assignment problems.

References


