Information Systems Analysis

# Temporal Logic and Timed Automata

(1)

## Introduction to temporal logic



*version 2.2*

# Program of lectures

1. Introduction to temporal logic
2. LTL logic
3. CTL logic
4. System model verification
5. UPPAAL timed automata
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7. NuSMV timed automata
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## Logic

- What is logic?
- Classical sentential logic
- Modal logic

# Contents of the lecture

## Time

- What is time?
- Dense and discrete time
- Finite and infinite time
- Linear, branching and parallel time
- Point and interval time

# Contents of the lecture

## Temporal logic

- What is temporal logic?
- Selected kinds of temporal logic
- Application of temporal logic

# Logic

- What is logic?
- Classical sentential logic
- Modal logic

# Logic

## What is logic?

- science of argumentation *(from Greek λογική /logiké/)*
- one of basic branches of the maths
  *(remaining are: the set theory and the arithmetic)*
- a mathematical description of reality
- a tool to distinguish between truth and falsehood
  *(as defined by Awerroes in XII century)*
- science of reasoning, and the art of reasoning
  *(as defined by Richard Whately in XIX century)*
- science of the most general laws of truth
  *(as defined by Gottlob Frege in XIX/XX century)*

# Logic

## Classical sentential logic

- is a set of logic laws;
- gives a logic state (*truth* / *falsity*) for every sentence,
  e.g. the sentence “October is a month.” is true;
- allows to reason about veracity* of a sentence,
  e.g. “If x and y are numbers, then their sum is a number too”.

* A sentence is true, if it describes an actual state of things.

# Logic

## Modal logic

- is the classical sentential logic mathematically specified.

Let AP be a set of atomic propositions

*An **Atomic proposition** is indivisible into other propositions.*
*E.g. "Water is wet." is not divisible into other sentences.*

A formula is generated by rules S1 and S2:

**S1:** every proposition $p \in$ AP is a formula;

**S2:** if $p$ and $q$ are formulas, then $p \wedge q$ and $\neg p$ are formulas too

(other classic logic connectives are also accessible).

# Modal logic

Basic and derivative logic relations:

| relation | description | meaning |
|---|---|---|
| conjunction | $p \wedge q$ | $p$ is true<br>and $q$ is true |
| negation | $\neg p$ | $p$ is not true |
| disjunction | $p \vee q$ | $\neg (\neg p \wedge \neg q)$ |
| implication | $p \Rightarrow q$ | $\neg p \vee q$ |
| equality | $p \Leftrightarrow q$ | $(p \Rightarrow q) \wedge (q \Rightarrow p)$ |

$p$ and $q$ are formulas

# Time

- What is time?
- Dense and discrete time
- Finite and infinite time
- Linear, branching and parallel time
- Point and interval time

# Time

## What is time?

- nonspatial continuum, in which events occur in an irreversible sequence from the past through the present to the future;
- one of the dimensions of the space–time continuum in physics.

Let $t_0$, $t_1$, $t_2 \in$ *time* be moments* of time.

** A **moment** is a zero-length time, represented e.g. by a point on a time axis.*

# Time

## Dense and discrete time

**Dense**: $(\forall t_0,t_1)\ (\exists t_2)\ (t_0 < t_2 < t_1)$

— for every two moments there exists a moment between them.

**Discrete**: $(\exists t_0,t_1)\ (\neg\exists t_2)\ (t_0 < t_2 < t_1)$

— there exist two such moments, that there does not exist any moment between them.

# Time

## Finite and infinite time

**Left–sided finite**: $(\exists t_0)\ (\neg\exists t_1)\ (t_1 < t_0)$

— the past is limited to some moment (e.g. the present one);
the future is infinite.

**Right–sided finite**: $(\exists t_0)\ (\neg\exists t_1)\ (t_1 > t_0)$

— the past is infinite;
the future is limited to some moment (e.g. the present one);

**Both–sided finite**: $(\exists t_0,t_1)\ (\neg\exists t_2)\ (t_2 < t_0 \vee t_2 > t_1)$

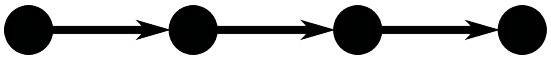
— the past and the future are limited to some moments;

**Infinite**: $(\forall t_0)\ (\exists t_1,t_2)\ (t_1 < t_0 < t_2)$

— the past and the future are unlimited.

# Time

## Linear, branching and parallel time

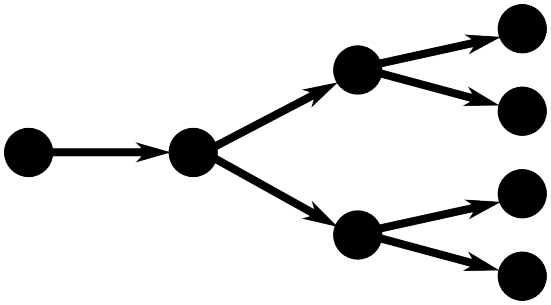


**Linear**: $(\forall t_0,t_1)\ (t_0 < t_1 \ \vee\ t_0 = t_1 \ \vee\ t_0 > t_1)$

— there exists only one variant of the flow of time.

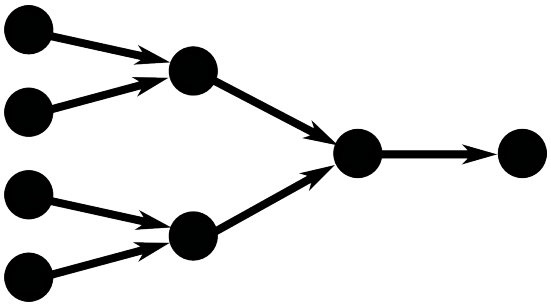
**Branching**:

— there exist different variants of the flow of time, having a common part with each other:



- left–sided linearity:

$(\forall t_0,t_1,t_2)\ (t_1< t_0 \wedge t_2< t_0 \Rightarrow t_1< t_2 \vee t_1= t_2 \vee t_1> t_2)$



- right–sided linearity:

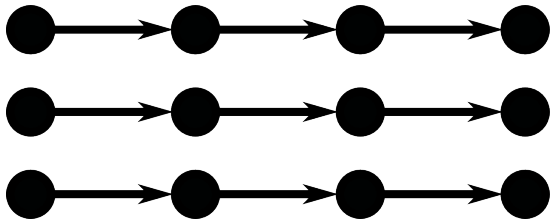
$(\forall t_0,t_1,t_2)\ (t_0< t_1 \wedge t_0< t_2 \Rightarrow t_1< t_2 \vee t_1= t_2 \vee t_1> t_2)$

# Linear, branching and parallel time

**Parallel**:

— there exist different variants of the flow of time, having no common part with each other;

— time is both left–sided and right–sided linear.



# Time

## Point and interval time

**Point**:

— the time structure consists of points only.

**Interval**:

— the time structure consists of intervals only.

# Temporal logic

- What is temporal logic?
- Selected kinds of temporal logic
- Application of temporal logic

# Temporal logic

## What is temporal logic?

- allows reasoning with time considered

  *(reasoning over time domain),*
- assigns the value *true* / *false* to modal logic propositions,
- places the value *true* / *false* in a time structure.

# Temporal logic

## Selected kinds of temporal logic:

- **LTL** (*Linear Temporal Logic*)

  — time: discrete, left-sided finite, linear, point-like;

- **CTL** (*Computation Tree Logic*)

  — time: discrete, left-sided finite, branching (left-sided linear), point-like;

- **RTCTL** (*Real-Time Computation Tree Logic*)

  — a version of CTL, where values of time are given quantitatively as constants;

- **PRTCTL** (*Parametrised Real-Time Computation Tree Logic*)

  — a version of CTL, where values of time are given quantitatively as variables.

# Temporal logic

## Selected kinds of temporal logic:

- **TCTL** (*Timed Computation Tree Logic*)

  — a version of CTL, where values of time are given quantitatively as constants;

- **CTL*** (*Full Branching Computation Tree Logic*)

  — a version of CTL of a greater expression, including expression of LTL;

- **ITL** (*Interval Temporal Logic*)

  — time: discrete, finite or infinite, linear, interval-like;

- **DC** (*Duration Calculus*)

  — a version of ITL, where truth of an expression is measured in a time interval.

# Application of temporal logic

- Mainly in modelling and verifying of systems that are:
  - concurrent

    (they consist of processes running in parallel, having access to common data, potentially interacting with each other);
  - reactive

    (they work in the action-reaction manner, in relation to arising circumstances, e.g.: controlling an electric reactor, an engine in a machine or the air traffic).

# The end

**Literature:**


- E.A. Emerson “Temporal and modal logic”, 1995
- R. Klimek “*Zastosowanie logiki temporalnej w specyfikacji i weryfikacji oprogramowania – w stronę czasu rzeczywistego”*, in: “Wykłady zaproszone. Systemy Czasu Rzeczywistego 2000”, 2001