Information Systems Analysis

Laboratories no. 5

version 5.0

Subject: Petri nets – generalized stochastic Petri nets.

Exercise 1. (5 pts)

To do: Create a generalized stochastic Petri net modeling a system of 3 logical gates *AND*, *NOR* and *XOR*, performing the function

$$\begin{cases} A \text{ AND } B = E \\ C \text{ NOR } D = F \\ E \text{ XOR } F = G \end{cases}$$

The net must contain:

- at least 7 places: A, B, C, D, E, F and G;
- immediate transitions that put tokens in these places;
- immediate transitions that remove tokens from places A, B, C, D and F.

Assume this net operates in the following order:

1. It determines the logical value of the inputs of the *AND* and *NOR* gates by placing tokens in the selected places A, B, C and D.
2. It determines the logical value of the input of the *XOR* gate by placing tokens in the places E and F if possible.
3. It determines the logical value of the output of the *XOR* gate by placing a token in the place G if possible.

Notice: A token in a given place (e.g. A) means true for that place (A = *true*).

Each place has a capacity of ∞, but can contain at most 1 token.

The above-mentioned sequence of the net operation is only an assumption for its simulation. Forcing this order by net structure will add 2 points to the score for this task.

Exercise 2. (5 pts)

To do: In laboratory no. 4, in exercise 2, a net for modeling traffic lights at a pedestrian crossing was created. Convert this net to a generalized stochastic Petri net, setting the type and parameters of transitions in order to best preserve their firing time and the sojourn time in the net markings.

Assume a finite waiting time for a pedestrian to press the button and calculate the probability of pressing it.

Notice: In the case of timed transitions, it is about their mean firing time and mean sojourn time.

Assume that the firing time of a transition in a Petri net with timed arcs, defined by the $[min; max]$ value range, is described by an exponential distribution, so that the mean firing time of the transition will be the lower value of this interval (but greater than 0 for $min = 0$) and use this value in the exercise.

Do not remove any existing net element (a place, a transition, an arc); also preserve their position.

Aid to the exercises

Generalized stochastic Petri net

The generalized stochastic Petri net is such a net, where:

- every transition is *immediate* or *timed*;
- immediate transitions have priority $\geqslant 0$;
- timed transitions have priority $= 0$ and an exponential random variable λ (λ_i for t_i), which may be dependent on net marking;
- there is no waiting for the immediate transition to fire (zero *firing time*);
- time that is defined by λ is needed to fire the timed transition;
- there exist inhibitor arcs too.

The firing time of a transition is the length of time from the moment the transition becomes available until the transition is fired.

The order of firing the transitions

The order of firing the transitions in a given net marking is as follows:

First – immediate transitions (if there are any available to fire) in order of their priority (from the highest) and their weights (from the highest, for transitions with equal priority) and in random order (for transitions with equal priority and equal weight).

Next – timed transitions (if there are any available to fire and no immediate transitions available to fire) in order of their λ parameter, where the greater λ_i is, the greater the probability of t_i to fire, so the smaller the average firing time t_i is.

Probability of firing a timed transition

The probability of firing a timed transition t_i in a marking M_j equals
$\frac{\lambda_i(M_j)}{\sum\limits_{t_k \in E(M_j)} \lambda_k(M_j)}$, where $E(M_j)$ is a set of transitions available to fire in M_j.

Mean firing time of a timed transition

The mean firing time of a timed transition t_i in a marking M_j equals $\frac{1}{\lambda_i(M_j)}$, where t_i is available to fire in M_j.

Mean sojourn time in a marking

The mean sojourn time in a marking M_j equals
$\frac{1}{\sum\limits_{t_i \in E(M_j)} \lambda_i(M_j)}$, where $E(M_j)$ is a set of transitions available to fire in M_j.

Recomended websites

- *Petri Nets: Properties, Analysis and Applications*
- *Generalized Stochastic Petri Nets: A Definition at the Net Level and Its Implications*