$$x'(t) = \underbrace{x(t)\cdot u(t)}_{f} \qquad x(0)=x_0$$

$$G(x,u) = \int_0^T \underbrace{(1-u(t))\,x(t)}_{g}\,dt$$

$$H(\lambda,x,u,t) = \lambda\, u(t)x(t) + (1-u(t))x(t) = x(t) + u(t)x(t)(\lambda-1)$$

MP:

$$H(\lambda,x,u^*,t) = \max_u \{x + ux(\lambda-1)\}$$

$x \geq 0$
$u \in (0,1)$

$$\downarrow$$

$$u^* = \begin{cases} 1 & \text{gdy } \lambda > 1 \\ 0 & \text{gdy } \lambda \leq 1 \end{cases}$$

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$$\begin{cases} \lambda' = -1 - u^*(\lambda-1) \\ \lambda(T) = 0 \end{cases} \qquad t \in (0,T)$$

$$\lambda(t) = \begin{cases} -t+T & \text{w odlk } t \in (T-1,\ T] \\ e^{T-1-t} & \text{w o.l. } t \in [0,\ T-1] \end{cases}$$

$$\downarrow$$

$$u^*(t) = \begin{cases} 1 & t \in [0,\ T-1) \\ 0 & t \in [T-1, T] \end{cases}$$