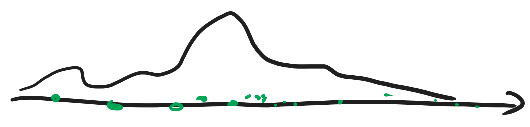
OBSERVATIONS

$x_1, x_2, \dots x_k, \dots x_N$ ⟶ $\hat f_N(x)$



KERNEL METHOD

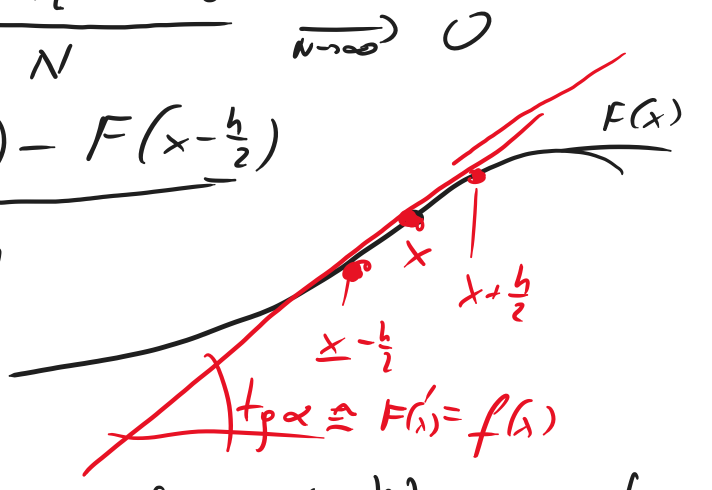
$\hat F_N(x) = \frac{\#\{x_k : x_k \le x\}}{N} = \frac{\sum_{k=1}^{N} I(x_k)}{N}$ $I(x_k) = \begin{cases} 1, & \text{as } x_k \le x \\ 0 & \text{as } x_k > x \end{cases}$

$\hat F_N(x) \xrightarrow{MSE} F(x)$ TRUE UNKNOWN

$E\hat F_N(x) = F(x)$ ⟶ bias $\hat F(x) = 0$

$Var\, \hat F_N(x) = \frac{F(x)[1-F(x)]}{N} \xrightarrow{N\to\infty} 0$

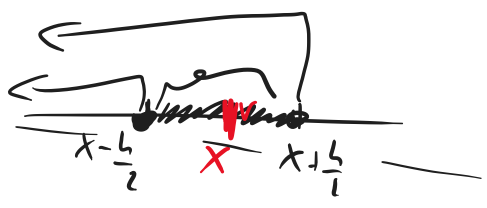
$f(x) = F'(x) = \lim_{h\to 0} \frac{F(x+\frac{h}{2}) - F(x-\frac{h}{2})}{h}$



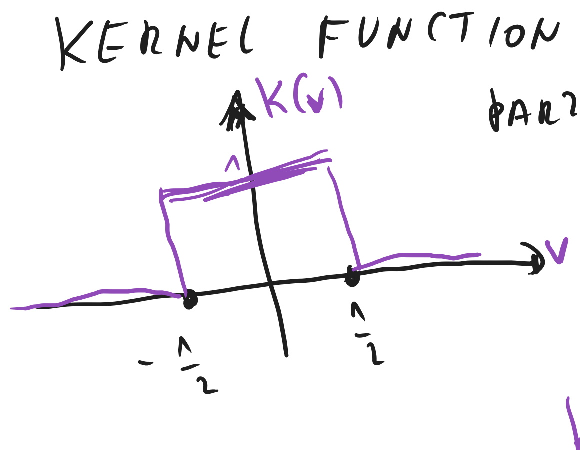
idea proposition

$\hat f_N(x) = \frac{\hat F_N(x+\frac{h}{2}) - \hat F_N(x-\frac{h}{2})}{h} = \frac{\frac{\#\{x_k : x_k \le x+\frac{h}{2}\}}{N} - \frac{\#\{x_k : x_k \le x-\frac{h}{2}\}}{N}}{h} =$

$= \frac{\#\{x_k : x-\frac{h}{2} \le x_k \le x+\frac{h}{2}\}}{Nh} = (\cancel{x}) = \frac{\sum_{k=1}^{N} K\left(\frac{x_k - x}{h}\right)}{Nh}$



KERNEL FUNCTION



PARZEN KERNEL (EXAMPLE)

$K(v) = \begin{cases} 1 & \text{as } |v| \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$K\left(\frac{x_k - x}{h}\right) \overset{\text{when?}}{==} 1$

$\left|\frac{x_k - x}{h}\right| \le \frac{1}{2}$

↓

$|x_k - x| \le \frac{1}{2} h$

$$\boxed{\hat f_N(x) = \frac{1}{Nh} \cdot \sum_{k=1}^{N} K\left(\frac{x_k - x}{h}\right)}$$

LIMIT PROPERTIES $(N \to \infty)$

HOW TO SET h ASYMPTOTICALLY?

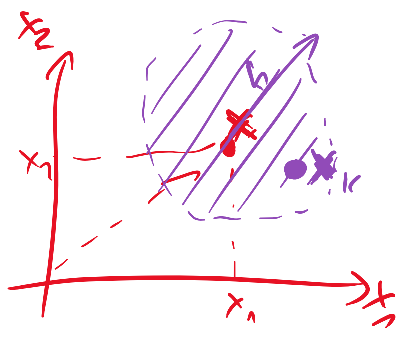
if $N \to \infty$ $h(N) \to 0$ then bias $\hat f_N(x) \to 0$

$Nh(N) \to \infty$ then var $\hat f(x) \to 0$

$h(N) \hat{=} \frac{1}{\sqrt{N}}$

IF $x \in R^d$ $f(x) = f(x_1, x_2)$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



$x_1, x_2, \dots, x_N$

$\hat f_N(x) = \frac{1}{Nh} \sum_{k=1}^{N} K\left(\frac{\|x_k - x\|}{h}\right)$

$x_k = \begin{bmatrix} x_{k,1} \\ x_{k,2} \end{bmatrix}$

$N \to \infty$ $h \to 0$

$Nh^d \to \infty$ STRONGER RESTRICTIVE CONDITION

$d = 2$ $Nh^2 \to 0$

$h \to 0$ much more slower then for $d=1$ case