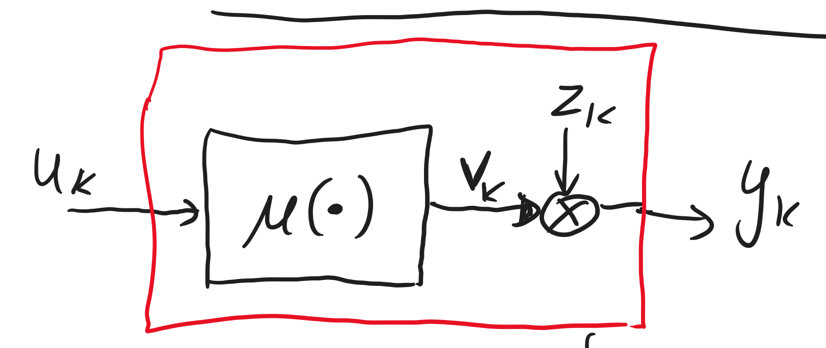
REGRESSION FUNCTION



z_k - random noise
$E z_k = 0$
$\mathrm{var}\, z_k < \infty$
z_k, u_k ← independent
v_k - hidden (not accessible)

OBSERVATIONS

$\{(u_k, y_k)\}_{k=1}^{N}$

DEF.

$$R(u) = E\{y_k \mid u_k = u\} \overset{\text{IN OUR PROBLEM}}{=} E\{\mu(u_k) + z_k \mid u_k = u\} =$$

$$= E\{\mu(u_k) \mid u_k = u\} + E\{z_k \mid u_k = u\} = \mu(u) + \underbrace{E z_k}_{0}$$

$$\boxed{R(u) = \mu(u)}$$

NONPARAMETRIC ESTIMATION OF REGRESSION FUNCTION – KERNEL METHOD

$f(u)$ – p.d.f.

ASSUME THAT

$f(u) > 0$

$$R(u) = \frac{R(u) \cdot f(u)}{f(u)} = \frac{g(u)}{f(u)}$$

$g(u) = R(u) \cdot f(u)$

$$\hat{R}(u) = \frac{\hat{g}(u)}{\hat{f}(u)} = \frac{\cancel{\frac{1}{Nh}} \sum_{k=1}^{N} y_k K\left(\frac{u_k - u}{h}\right)}{\cancel{\frac{1}{Nh}} \sum_{k=1}^{N} K\left(\frac{u_k - u}{h}\right)}$$

$$\boxed{\hat{\mu}(u) = \hat{R}(u) = \frac{\sum_{k=1}^{N} y_k K\left(\frac{u_k - u}{h}\right)}{\sum_{k=1}^{N} K\left(\frac{u_k - u}{h}\right)}}$$

KRE

NONPARAMETRIC ESTIMATION OF THE REGRESSION FUNCTION – ORTHOGONAL EXPANSION METHOD

$$\mu(u) = R(u) = \frac{\overset{\text{output}}{\overbrace{R(u) \cdot f(u)}}}{f(u)} = \frac{g(u)}{f(u)} = \frac{\sum_{i=1}^{\infty} b_i \varphi_i(u)}{\sum_{i=1}^{\infty} a_i \varphi_i(u)}$$

$f(x) \in L^2$

$a_i = E \varphi_i(u_k)$ $\qquad$ $b_i = E\, y_k \varphi_i(u_k)$

ESTIMATOR:

$$\hat{a}_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(u_k) \qquad \hat{b}_i = \frac{1}{N} \sum_{k=1}^{N} y_k \varphi_i(u_k)$$

$$\hat{R}(u) = \frac{\hat{g}(u)}{\hat{f}(u)} = \frac{\sum_{i=1}^{S} \hat{b}_i \varphi_i(u)}{\sum_{i=1}^{S} \hat{a}_i \varphi_i(u)}$$