INSTRUMENTAL VARIABLES

$$Y_N = \bar{\Phi}_N a^* + Z$$

LS: $\hat{a}_{LS} = \left(\bar{\Phi}_N^T \bar{\Phi}_N\right)^{-1} \bar{\Phi}_N^T Y_N$

IV: $\hat{a}_{IV} = \left(\frac{1}{N}\Psi_N^T \bar{\Phi}_N\right)^{-1} \frac{1}{N}\Psi_N^T Y_N$ ← GENERALIZATION OF LS $\Psi_N = \bar{\Phi}_N \rightarrow LS$ SPECIAL CASE

additional matrix of instrumental variables

$\dim \Psi_N = \dim \bar{\Phi}_N^T$

POSTULATES:

1° $\frac{1}{N}\Psi_N^T \bar{\Phi}_N = \frac{1}{N}\sum_{k=1}^{N} \psi_k \phi_k^T \longrightarrow E\psi_1\phi_1$

THIS LIMIT SHOULD BE INVERTIBLE

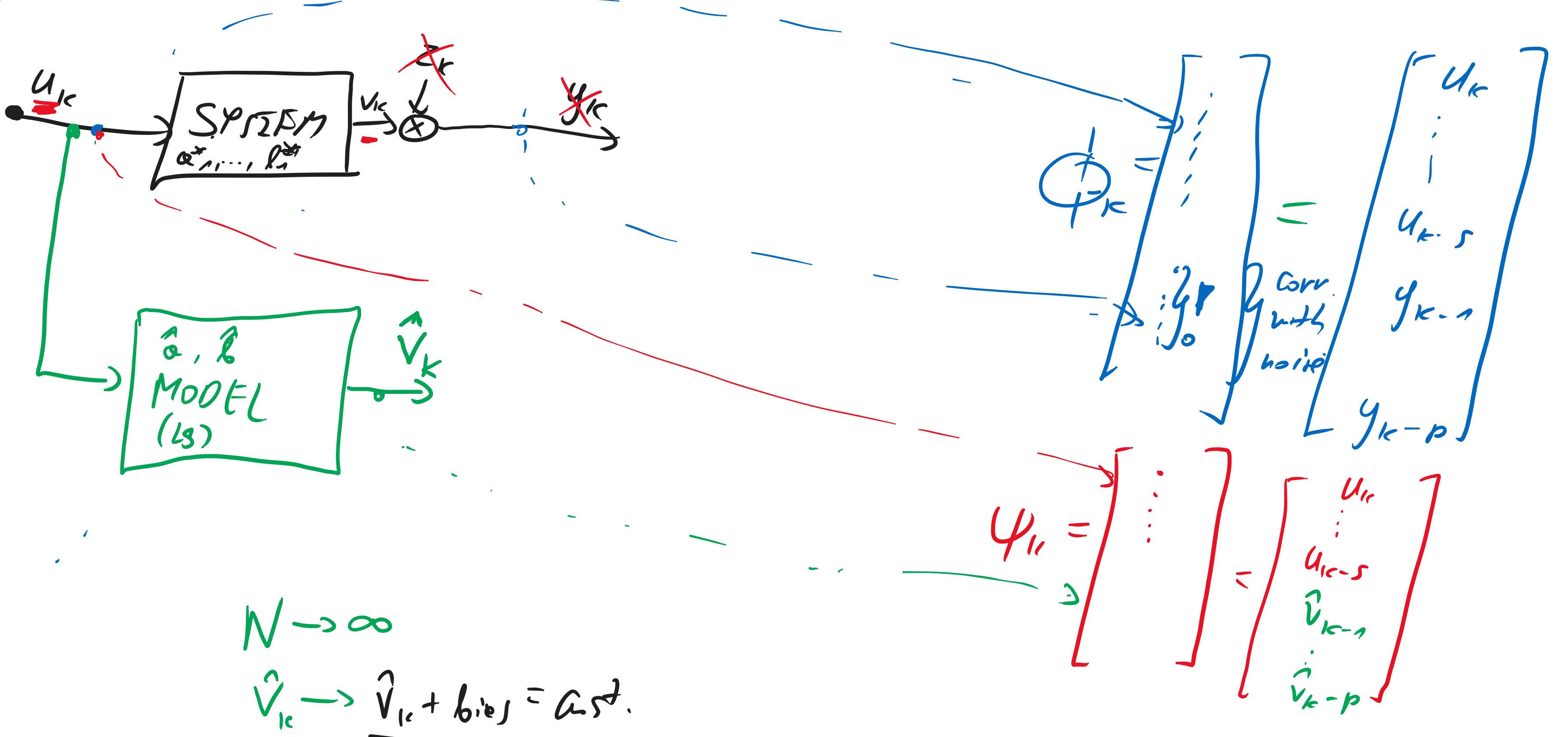
ψ_k MUST BE CORRELATED WITH ϕ_k

2° $\frac{1}{N}\Psi_N^T Z_N = \frac{1}{N}\sum_{k=1}^{N} \psi_k z_k \longrightarrow E\psi_1 z_1 = 0$

ψ_k MUST **NOT** BE CORRELATED WITH z_k

$$E\psi_1 z_1 = E\psi_k \cdot \underset{}{\overset{0}{E z_k}} = 0$$

THE IDEA



$N \rightarrow \infty$

$\hat{V}_k \rightarrow \underline{\hat{V}_k + \text{bias}} = \text{cst.}$

THEOREM

IF THE POSTULATES 1° AND 2° ARE FULFILLED THEN

$$\hat{a}_{I.V.} \xrightarrow[N \to \infty]{p.1.} a^*$$