Nonlinear system identification under various prior knowledge

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Approaches to system identification

- parametric (traditional)
- nonparametric
- parametric-nonparametric (combined)
- semiparametric

Static nonlinearity

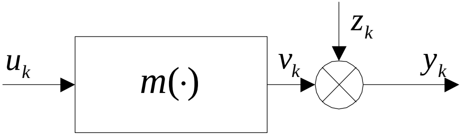


Figure: Static nonlinear element

$$R(u) = E\{y_k | u_k = u\} = E\{m(u_k) + z_k | u_k = u\} = m(u)$$

Two kinds of knowledge

1. parametric, (shape of formula describing $m(u, c^*) = m(u)$)

$$m(u,c) = c_1 f_1(u) + c_2 f_2(u) + \ldots + c_p f_p(u)$$

or

$$m(u,c) = f_1(u,c_1) \circ f_2(u,c_2) \circ \ldots \circ f_p(u,c_p)$$

e.g.

$$m(u,c) = c_1 + c_2 u + c_3 u^2 \qquad \text{or} \qquad m(u,c) = c_1(\sin c_2 u + c_3 e^{c_4 u})$$

2. non-parametric, (measurements)

$$\{(u_k, y_k)\}_{k=1}^{N}$$

The classical approach (parametric)

$$\widehat{c}_N = \arg\min_c \sum_{k=1}^{N} (y_k - m(u_k, c))^2$$

Features:

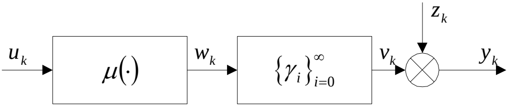
- fast convergence to the optimal model in the class $m(u,c)$ (under rich a priori knowledge)
- but, the risk of systematic approximation error when the model is bad
- complicated and badly conditioned computations (linear or nonlinear least squares)

Nonparametric estimates

Orthogonal expansion, kernel regression

- are based on the measurements only
- does not involve the unknown characteristic must belong to the finite dimmensional class ($p \longrightarrow \infty$ as $N \longrightarrow \infty$)
- converge to the true characteristic
- are computationally simple
- have more deegrees of freedom (the choice of tunning parameters and basis functions)

Hammerstein system



$$R(u) = E\{y_k | u_k = u\} = E\left\{\sum_{i=0}^{\infty} \gamma_i \mu(u_{k-i}) + z_k | u_k = u\right\} = \gamma_0 \mu(u) + \zeta$$

Nonparametric algorithms

Assumptions

- The nonlinear characteristic $m(u)$ can be an *arbitrary* function, square integrable on $[-1,1]$, and *e.g.*:
  - differentiable
  - continuous
  - piecewise-smooth
- There is a set of *sorted* input-output measurements $\{u_l, y_l\}$, $l = 1, \ldots, k$.

Remark

The class of admissible characteristic is now so ample that it **cannot** *be represented by any parametric model*

Orthogonal series basics

Observation

Any nonlinearity $\mu(u)$ has its orthogonal expansion

$$\begin{aligned}\mu(u) &= \alpha_0 + \alpha_1 \cdot p(u) + \cdots + \alpha_K \cdot p_K(u) + \cdots \textit{ ad infinitum} \\ &= \sum_{i=0}^{\infty} \alpha_i \cdot p_i(u)\end{aligned}$$

where $\{p_i\}, i = 0, 1, \ldots$ is an orthogonal basis in $L^2[-1,1]$ and

$$\alpha_i = \langle \mu, p_i \rangle = \int_{-1}^{1} \mu(u) \cdot p_i(u)\, du$$

Orthogonal series estimate

A generic orthogonal series algorithm

An orthogonal series algorithm has a form

$$\hat{\mu}(u) = \sum_{i=0}^{K(k)} \hat{\alpha}_i \cdot p_i(u)$$

where $K(k)$ is an non-decreasing number sequence and

$$\hat{\alpha}_i = \sum_{l=1}^{k} \int_{u_{l-1}}^{u_l} y_l \cdot p_i(u)\, du$$

MISE error

The performance of the algorithm is measured by a mean integrated square error

$$\text{MISE}\,\hat{\mu} = E\int_{-1}^{1}\left(\mu\left(u\right)-\hat{\mu}\left(u\right)\right)^2 du$$

Error decomposition

$$\text{MISE}\,\hat{\mu} = \overbrace{\sum_{i=K(k)+1}^{\infty}\alpha_i^2}^{\text{approx}^2\,\mu^K\text{ - deterministic error}} + \overbrace{\overbrace{\sum_{i=0}^{K(k)}\text{bias}^2\hat{\alpha}_i}^{\text{bias}^2\,\hat{\mu}} + \overbrace{\sum_{i=0}^{K(k)}\text{var}\,\hat{\alpha}_i}^{\text{var}\,\hat{\mu}}}^{\text{stochastic errors}}$$

Example I – Legendre polynomial series estimate

Legendre polynomial estimate

Legendre polynomial basis is *recursively* defined as

$$p_i(u) = \sqrt{\frac{2i+1}{2}} \cdot P_i(u)$$

where

$$P_i(u) = \frac{2i-1}{i} \cdot xP_{i-1}(u) - \frac{i-1}{i} \cdot P_{i-2}(u)$$

with

$$P_1(u) = u \text{ and } P_0(u) = 1$$

Example II – Chebyshev polynomial series estimate

Chebyshev polynomial estimate

Chebyshev polynomial basis is *recursively* defined as

$$p_i(u) = \sqrt{\frac{1}{1-u^2}} \cdot P_i(u)$$

where

$$P_i(u) = 2xP_{i-1}(u) - P_{i-2}(u)$$

with

$$P_1(u) = u \text{ and } P_0(u) = 1$$

Convergence & rates

Convergence

If $K(k) \to \infty$ and $K(k)/k \to 0$ then

$$\mathrm{MISE}\,\hat{\mu} \to 0 \text{ as } k \to \infty.$$

Convergence rate

Let λ be a number of derivatives of μ. If $K(k) = k^{\frac{1}{2\lambda+1}}$ then

$$\mathrm{MISE}\,\hat{\mu} \sim k^{-\frac{2\lambda}{2\lambda+1}}.$$

- the smoother nonlinearity the faster convergence
- the rate can be established for smooth nonlinearities only

Example III - Wavelet series estimate

Any nonlinearity can be represented in a multiresolution form

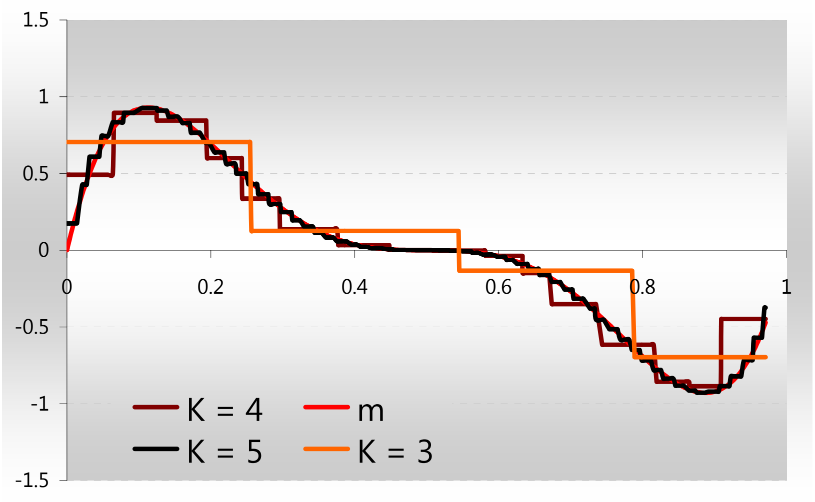
$$\mu(u) = \overbrace{\sum_{n=0}^{2^M-1} \alpha_{Mn} \cdot \varphi_{Mn}(u)}^{\text{A 'crude' approximation}} + \overbrace{\sum_{n=0}^{2^M-1} \beta_{Mn} \cdot \psi_{Mn}(u)}^{\text{details at the resolution } 2^M} + \cdots$$

$$+ \overbrace{\sum_{n=0}^{2^{K-1}-1} \beta_{K-1,n} \cdot \psi_{K-1,n}(u)}^{\text{details at the resolution } 2^{K-1}} + \cdots \textit{ ad infinitum}$$

where

$$\alpha_{Mn} = \int_{-1}^{1} \mu(u) \cdot \varphi_{Mn}(u)\, du \text{ and } \beta_{mn} \int_{-1}^{1} \mu(u) \cdot \psi_{mn}(u)\, du$$

Example III - multiresolution approximation



Example III - Wavelet series estimate

A generic wavelet estimate

The wavelet estimate is of the form

$$\hat{\mu}(u) = \sum_{n=0}^{2^M-1} \hat{\alpha}_{Mn} \cdot \varphi_{Mn}(u) + \sum_{m=M}^{K(k)-1} \sum_{n=0}^{2^m-1} \hat{\beta}_{mn} \cdot \psi_{mn}(u)$$

where

$$\hat{\alpha}_{Mn} = \sum_{l=1}^{k} y_l \cdot \int_{u_{l-1}}^{u_l} \varphi_{Mn}(u)\, du \text{ and } \hat{\beta}_{mn} = \sum_{l=1}^{k} y_l \cdot \int_{u_{l-1}}^{u_l} \psi_{mn}(u)\, du$$

- $\varphi(u)$ and $\psi(u)$ can be from *Haar* or *Cohen-Daubechies-Vial* family...

Convergence & rates

Convergence and its rate

If $K(k) \to \infty$ and $2^{K(k)}/k \to 0$ then

$$\text{MISE}\,\hat{\mu} \to 0 \text{ as } k \to \infty.$$

Let λ be a number of derivatives of μ. If $K(k) = \frac{1}{2\lambda+1}\log_2 k$ then

$$\text{MISE}\,\hat{\mu} \sim k^{-\frac{2\lambda}{2\lambda+1}}$$

Let $\mu(u)$ has a finite number of jumps. If $K(k) = \frac{1}{2}\log_2 k$ then

$$\text{MISE}\,\hat{\mu} \sim k^{-\frac{1}{2}}$$

- the smoother nonlinearity the faster convergence
- the rate can be established also for **discontinuous** nonlinearities!

Kernel estimate

A generic kernel estimate

The algorithm based on kernels has the generic form

$$\hat{\mu}(u) = \frac{\sum_{l=1}^{k} y_l \cdot K\left(\frac{u-u_l}{h(k)}\right)}{\sum_{l=1}^{k} K\left(\frac{u-u_l}{h(k)}\right)}$$

where K is so called *kernel function*.

Example – rectangular kernel

Rectangular kernel

Using rectangular (uniform) kernel, $K(u) = I_{[0,1)}(u)$, we obtain a simple estimate

$$\hat{\mu}(u) = \frac{\sum_{l \in L} y_l}{\#L} \text{ where } L = \{l : u_l \in [u - h(k), u + h(k))\}$$

- Other kernels: *Epanechnikov, Gauss, Cauchy...*

Convergence & rates

Convergence

If $h(k) \to 0$ and $k \cdot h(k) \to \infty$ then

$$\text{MISE}\,\hat{\mu} \to 0 \text{ as } k \to \infty.$$

Convergence rate

Let λ be a number of derivatives of μ. If $h(k) = k^{-\frac{1}{2\lambda+1}}$ then

$$\text{MISE}\,\hat{\mu} \sim k^{-\frac{2\lambda}{2\lambda+1}}.$$

- the smoother nonlinearity the faster convergence
- the rate can be established for smooth nonlinearities only

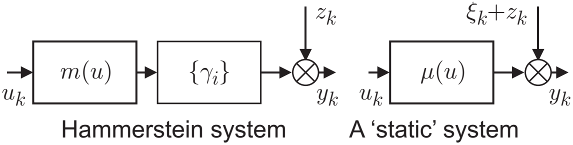
Class of systems

The class of systems to which the above algorithms can be **directly** includes many popular structures like:

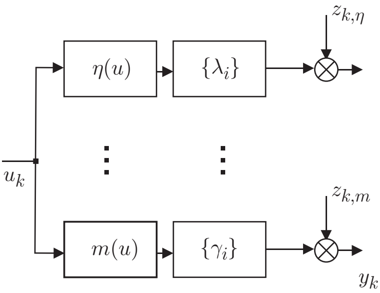
- *Hammerstein* system,
- parallel system,
- *Uryson* system, etc.

For instance, for *Hammerstein* system we have

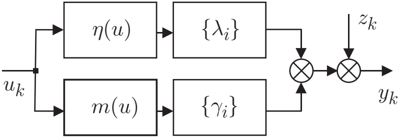
$$y_k = \overbrace{\gamma_0 m\left(u_k\right)}^{\mu(u_k)} + \overbrace{\sum_{i=1}^{\infty} \gamma_i \left[m\left(u_{k-i}\right) - Em\left(u_1\right)\right]}^{\xi_k} + z_k = \mu\left(u_k\right) + \xi_k + z_k$$



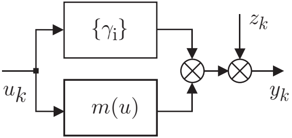
Examples of admissible dynamic nonlinear systems



Multichannel system



Uryson system



Parallel system

A censored (kernel) sample-mean approach to Wiener system identification

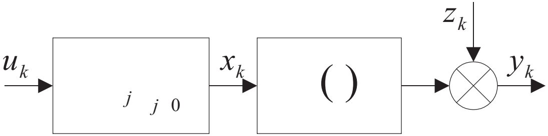


Figure: Wiener system

$$y_k = \mu \left(\sum_{j=0}^{\infty} \lambda_j u_{k-j} \right) + z_k$$

Assumptions

(A1) $\{u_k\}$ – i.i.d., bounded ($|u_k| < u_{\max}$) random process
(A1a) there exists a p.d.f. of the input $\vartheta_u(u_k)$, which is a continuous and strictly positive function in the estimation points x, i.e., $\vartheta_u(x) > 0$. or
(A1b) It holds that $P(u_k = x) > 0$ if u_k has discrete distribution.
(A2) The unknown impulse response $\{\lambda_j\}_{j=0}^{\infty}$ of the linear *IIR* filter is bounded from above as follows

$$|\lambda_j| \leqslant c \cdot \lambda^j$$

where $\lambda \in (0, 1)$ is a priori known constant.
(A3) The nonlinearity $\mu(x)$ is an arbitrary function, which is continuous almost everywhere on $x \in (-u_{\max}, u_{\max})$ (in the sense of Lebesgue measure).
(A4) The output noise $\{z_k\}$ is a zero-mean ergodic process, which is independent of the input $\{u_k\}$.

The algorithm

$$\widehat{\mu}_N(x) = \frac{\sum_{k=1}^{N} y_k \cdot K\left(\frac{\Delta_k(x)}{h(N)}\right)}{\sum_{k=1}^{N} K\left(\frac{\Delta_k(x)}{h(N)}\right)}$$

where

$$\begin{aligned} \Delta_k(x) \triangleq \; & |u_k - x|\,\lambda^0 + |u_{k-1} - x|\,\lambda^1 + |u_{k-2} - x|\,\lambda^2 + \ldots \\ & \ldots + \left|u_{k-S(N)-1} - x\right|\,\lambda^{S(N)-1} \end{aligned}$$

Parametric-nonparametric approach to Hammerstein system identification

Assumptions

A1: $|u_k| \leqslant u_{\max}$, $\exists$ p.d.f. $\nu(u)$

A2:

$$|\mu(u)| \leqslant w_{\max}$$

A3:

$$\sum_{i=0}^{\infty} |\gamma_i| < \infty$$

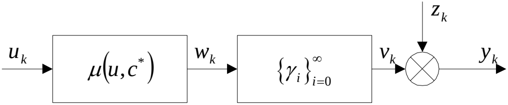
A4: $\mu(u_0)$ known for some u_0 (let $u_0 = 0$) and $\gamma_0 = 1$

Z5:

$$z_k = \sum_{i=0}^{\infty} \omega_i \varepsilon_{k-i}$$

$\{\varepsilon_k\}$ – i.i.d., independent of $\{u_k\}$, $E\varepsilon_k = 0$, $|\varepsilon_k| \leqslant \varepsilon_{\max}$
$\{\omega_i\}_{i=0}^{\infty}$ – unknown,
$\sum_{i=0}^{\infty} |\omega_i| < \infty$

Parameter knowledge



- we are given the formula $\mu(u,c)$, such that $\mu(u,c^*) = \mu(u)$, where $c^* = (c_1^*, c_2^*, ..., c_m^*)^T$ – true parameters
- $\mu(u,c)$ – differentiable with respect to c
- for each $u \in [-u_{\max}, u_{\max}]$

$$\|\nabla_c \mu(u,c)\| \leqslant G_{\max} < \infty, \qquad c \in \mathcal{C}(c^*)$$

- c^* is identifiable, i.e. there exist such sequence $\overline{u}_1, \overline{u}_2, ..., \overline{u}_{N_0}$ that

$$\mu(\overline{u}_n, c) = \mu(\overline{u}_n, c^*),\ n = 1,2,...,N_0 \Rightarrow c = c^*$$

Estimation of the static characteristic

$$Q_{N_0}(c) = \sum_{n=1}^{N_0} \left[w_n - \mu(\overline{u}_n, c)\right]^2 \qquad c^* = \arg\min_c Q_{N_0}(c)$$

Stage 1: On the basis on M pairs $\{(u_k, y_k)\}_{k=1}^{M}$, for N_0 fixed points $\{\overline{u}_n;\ n = 1, 2, ..., N_0\}$ estimate $\{w_n = \mu(\overline{u}_n, c^*);\ n = 1, 2, ..., N_0\}$

$$\widehat{w}_{n,M} = \widehat{R}_M(\overline{u}_n) - \widehat{R}_M(0)$$

Stage 2: Optimize the following criterion

$$\widehat{Q}_{N_0,M}(c) = \sum_{n=1}^{N_0} \left[\widehat{w}_{n,M} - \mu(\overline{u}_n, c)\right]^2$$

with respect to c and take the $\widehat{c}_{N_0,M}$ as the estimate of c^*.

Limit properties

If the system is identifiable then

$$\delta \cdot \|c - c^*\|^2 \leqslant Q_{N_0}(c) \leqslant D \cdot \|c - c^*\|^2$$

Theorem

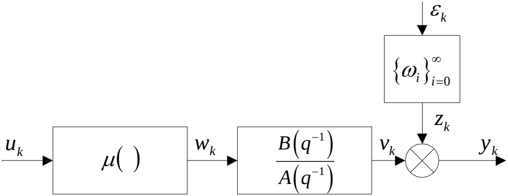
Assume that $\widehat{c}_{N_0,M}$ is unique and $\widehat{c}_{N_0,M}$, $c^* \in C$ for each M, where C is bounded convex set in R^m. If in stage 1

$$\widehat{R}_M(\overline{u}_n) = R(\overline{u}_n) + O(M^{-\tau}) \text{ in probability as } M \to \infty$$

for $n = 1, 2, ..., N_0$ and for $\overline{u}_n = 0$ then

$$\widehat{c}_{N_0,M} = c^* + O(M^{-\tau}) \text{ in probability as } M \to \infty$$

Prior knowledge of the linear dynamics



$$
\begin{aligned}
v_k &= b_0 w_k + ... + b_s w_{k-s} + a_1 v_{k-1} + ... + a_p v_{k-p} \\
\theta &= (b_0, b_1, ..., b_s, a_1, a_2, ..., a_p)^T \\
\vartheta_k &= (w_k, w_{k-1}, ..., w_{k-s}, y_{k-1}, y_{k-2}, ..., y_{k-p})^T \\
y_k &= \vartheta_k^T \theta + \bar{z}_k, \qquad \bar{z}_k = z_k - a_1 z_{k-1} - ... - a_p z_{k-p} \\
Y_N &= \Theta_N \theta + Z_N, \quad \Theta_N = (\vartheta_1, ..., \vartheta_N)^T, \quad Z_N = (\bar{z}_1, ..., \bar{z}_N)^T
\end{aligned}
$$

Nonparametric instrumental variables

$$\widehat{\theta}_{N,M}^{(IV)} = (\widehat{\Psi}_{N,M}^T \widehat{\Theta}_{N,M})^{-1} \widehat{\Psi}_{N,M}^T Y_N$$

where

$$\begin{aligned}
\widehat{\Theta}_{N,M} &= (\widehat{\vartheta}_{1,M}, \dots, \widehat{\vartheta}_{N,M})^T \\
\widehat{\vartheta}_{k,M} &= (\widehat{w}_{k,M}, \dots, \widehat{w}_{k-s,M}, y_{k-1}, \dots, y_{k-p})^T \\
\widehat{\Psi}_{N,M} &= (\widehat{\psi}_{1,M}, \dots, \widehat{\psi}_{N,M})^T \\
\widehat{\psi}_{k,M} &= (\widehat{w}_{k,M}, \dots, \widehat{w}_{k-s,M}, \widehat{w}_{k-s-1,M}, \dots, \widehat{w}_{k-s-p,M})^T
\end{aligned}$$

Limit properties (1)

Theorem

If the estimate $\widehat{R}_M(u)$ *is bounded, converges to* $R(u)$*, and the estimation error in the points* $u \in \{0, u_{k-r}$*; for* $k = 1, 2, ..., N$ *and* $r = 0, 1, ..., s+p\}$ *behaves like*

$$\left|\widehat{R}_M(u) - R(u)\right| = O(M^{-\tau}) \text{ in probability}$$

then for $NM^{-\tau} \longrightarrow 0$ *the following conditions are fulfilled*

(a') $Plim_{M,N\rightarrow\infty}\left(\frac{1}{N}\widehat{\Psi}^T_{N,M}\widehat{\Theta}_{N,M}\right)$ *exists and is not singular*

(b') $Plim_{M,N\rightarrow\infty}\left(\frac{1}{N}\widehat{\Psi}^T_{N,M}Z_N\right) = 0$

Limit properties (2)

Theorem

Under assumptions of Theorem 2 it holds that

$$\widehat{\theta}^{(IV)}_{N,M} \longrightarrow \theta \quad \textit{in probability}$$

as $N, M \longrightarrow \infty$, *if* $NM^{-\tau} \longrightarrow 0$. *In particular, for* $M \sim N^{(1+\alpha)/\tau}$, $\alpha > 0$, *the asympptotic rate of convergencehas the form*

$$\left\| \widehat{\theta}^{(IV)}_{N,M} - \theta \right\| = O(N^{-\min(\frac{1}{2},\alpha)}) \quad \textit{in probability}$$

Optimal instrumental variables

$$\Delta_N^{(IV)}(\Psi_N) \triangleq \widehat{\theta}_N^{IV} - \theta^*$$

$$Q(\Psi_N) \triangleq \max_{\left\|Z_N^*\right\|_2 \leq 1} \left\|\Delta_N^{(IV)}(\Psi_N)\right\|_2^2$$

$$Z_N^* \triangleq \frac{\frac{1}{\sqrt{N}} Z_N}{\bar{z}_{\max}}$$

Theorem

In Hammerstein system, for each admissible Ψ_N itholds that

$$\lim_{N\to\infty} Q(\Psi_N) \geqslant \lim_{N\to\infty} Q(\Psi_N^*) \text{ with probability } 1$$

where

$$\Psi_N^* = (\psi_1^*, \psi_2^*, ..., \psi_N^*)^T, \qquad \psi_k^* = (w_k, ..., w_{k-s}, v_{k-1}, ..., v_{k-p})^T.$$

Approximate realization

$$\widehat{\psi}^*_{k,M} = (\widehat{w}_{k,M}, \widehat{w}_{k-1,M}, \ldots, \widehat{w}_{k-s,M}, \widehat{v}_{k-1,M}, \widehat{v}_{k-2,M}, \ldots, \widehat{v}_{k-p,M})^T$$

$$\widehat{v}_{k,M} = \sum_{i=0}^{F} \widehat{\gamma}_{i,M} \widehat{w}_{k-i,M}$$

$$\widehat{\gamma}_{i,M} = \widehat{\varkappa}_{i,M} / \widehat{\varkappa}_{0,M}, \qquad \widehat{\varkappa}_{i,M} = \frac{1}{M} \sum_{k=1}^{M-i} (y_{k+i} - \overline{y})(u_k - \overline{u})$$

Summary

- Consistent estimates in the presence of colored noise
- Problem decomposition with use of nonparametric methods
- Broad class of models (non-linear-in-parameters + IIR)

Semiparametric algorithm – assumptions

- The nonlinear characteristic $m(u)$ can be an *arbitrary* function, square integrable, and *e.g.*:
  - differentiable
  - continuous
  - piecewise-smooth
- There is a set of input-output measurements $\{u_l, y_l\}$, $l = 1, \ldots, k$.
- There is a **polynomial** model $\mu_p(u)$ of order $p-1$ of the nonlinearity $\mu(u)$; *e.g.* hard-wired, or taken from *Matlab* System Identification toolbox.

Remark

The model can offer only crude approximations when the genuine nonlinearity turns out to be e.g. a piecewise smooth function with discontinuities.

Additive regression

- Having, by assumption, the polynomial model $\mu_p(u)$, we are interested in the remaining part:

$$\mu_r(u) = \mu(u) - \mu_p(u) = E(y_k | u_k = u) - \mu_p(u)$$

  which will further be referred to as *residual nonlinearity*.

- The polynomial model $\mu_p(u)$ can *exactly* be represented as a 'crude' wavelet approximation

$$\mu_p(u) = \sum_{i=0}^{p-1} \alpha_i \cdot u^i = \sum_{n=0}^{2^M-1} \alpha_{Mn}^p \cdot \varphi_{Mn}(u)$$

  where $\alpha_{Mn}^p = \langle \tilde{\mu}_p, \varphi_{Mn} \rangle$.

Wavelet estimate of a residual function

- The estimate is a version of the presented wavelet estimate

$$\hat{\mu}_r(u) = \sum_{n=0}^{2^M-1} \hat{\alpha}_{Mn} \cdot \varphi_{Mn}(u) + \sum_{m=M}^{K-1} \sum_{n=0}^{2^m-1} \hat{\beta}_{mn} \cdot \psi_{mn}(u)$$

  where the expansion coefficient estimates are computed in a convenient *on-line* fashion

$$\begin{bmatrix} \hat{\alpha}_{Mn}^{(k+1)} \\ \hat{\beta}_{mn}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_{Mn}^{(k)} \\ \hat{\beta}_{mn}^{(k)} \end{bmatrix} + (y_{k+1} - y_{l+1}) \begin{bmatrix} \Phi_{Mn}(u_{k+1}) - \Phi_{Mn}(u_l) \\ \Psi_{mn}(u_{k+1}) - \Psi_{mn}(u_l) \end{bmatrix}$$

  where $\Phi_{Mn}(u)$ and $\Psi_{mn}(u)$ are antiderivatives of $\varphi_{Mn}(u)$ and $\psi_{mn}(u)$.
- The algorithm starts with

$$\begin{bmatrix} \hat{\alpha}_{Mn}^{(1)} \\ \hat{\beta}_{mn}^{(1)} \end{bmatrix} = \begin{bmatrix} -\alpha_{Mn}^{p} \\ 0 \end{bmatrix} \text{ and } \{(u_0 = 0, y_0 = 0), (u_1 = 1, y_1 = 0)\}.$$

Convergence & rates

Convergence rate

Let λ be a number of derivatives of μ. If $K(k) = \frac{1}{2\lambda+1}\log_2 k$ then

$$\operatorname{MISE}\hat{\mu} \sim k^{-\frac{2\lambda}{2\lambda+1}}$$

Let $\mu(u)$ has a finite number of jumps. If $K(k) = \frac{1}{2}\log_2 k$ then

$$\operatorname{MISE}\hat{\mu} \sim k^{-\frac{1}{2}}$$

- the smoother nonlinearity the faster convergence (the **same** as for polynomials)
- the rate can be established also for **discontinuous** nonlinearities!
- the convergence holds regardless the actual type of the pre-model $\mu_p(u)$, be it regular or orthogonal.

Example - Legendre polynomial model and Haar wavelet amendment

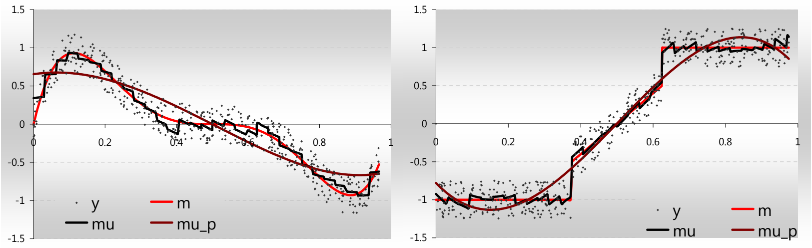
- The nonlinearities

$$m(u) = 5(u^5 - u^3) \text{ and } m(u) = \begin{cases} -1 & \text{if} \quad u < 3/8 \\ 4x - 2 & \text{if} \quad 3/8 \leq u < 5/8 \\ 1 & \text{if} \quad 5/8 \leq u \end{cases}$$

- The model (based on *Legendre* polynomial of order $p = 4$)

$$\mu_p(u) = \sum_{i=0}^{4} \alpha_i p_i(u) \text{ where } \alpha_i = \langle \mu_n, p_i \rangle$$

Example – simulation results



Final conclusions

- *Parametric* and *nonparametric* algorithms complete each other rather than compete…
- The choice of the algorithm type can separately be made appropriately to a different *a priori* knowledge available for either of the system block.
- The convergence of the algorithms can formally be shown for *virtually all* nonlinear characteristics.
- *Semiparametric* algorithms benefit from advantages of parametric and nonparametric ones.

A discovery of Ceres

Beginnings...

- Ceres was spotted by *G. Piazzi* as a result of an exhaustive search in an attempt to verify *Titius-Body* rule (**ad hoc model**) governing the distance of the Solar system objects from Sun).
- The observation of its position were recorded yet no orbit parameters had been established.
- The dwarf-planet was lost after traversing behind Sun.
- Several astronomers (*Body*, *von Zach*, *Olbers*) tried to determine the orbit and failed...

A discovery of Ceres

Towards better models. . .

- They used **a wrong model** (inappropriate *a priori* knowledge) assuming circular shape of the orbit (which result in *a biased model with systematic error*), and did also not correctly deal with error in measurements.
- Gauss ingeniously took into account these errors (proposing his *least squares* algorithm to cope with *random errors*) but also used a **better model** admitting elliptical orbits (*e.g.* the one based on *Kepler*'s laws).
- That the *Kepler*'s laws were not an **ultimate model** for celestial bodies motion was discovered and explained another 100 years later by another genius, *Albert Einstein*, whose *general relativity theory* finally explained Mercury's orbit anomalies.

Selected recent papers of the team

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