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Multi-level Identification of Hammerstein-Wiener (N-L-N) System in Active Experiment

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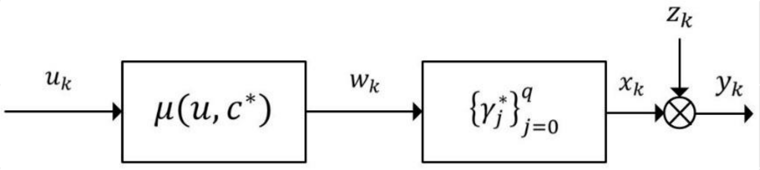
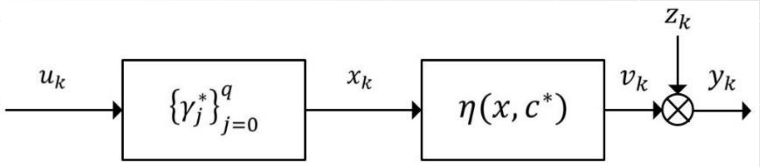
ICINCO, Porto, 31 VII 2018

Outline

1) Context of block-oriented models
2) Statement of the problem
3) General assumptions
4) The identification algorithm
    a) Identification of dynamic linear block
    b) Extraction of second nonlinear characteristic
    c) Output signal filtration
    d) Identification of first nonlinear characteristic
5) Simple simulation example
6) Applications
7) Conclusions

Context of block-oriented models





Fundamental blocks:

Linear dynamic (L/LD)

Nonlinear static (N/NS)

Relevant structures:

Hammerstein model

Wiener model

Cascade structures

Elementary identification methods:

Least squares method

Kernel estimates

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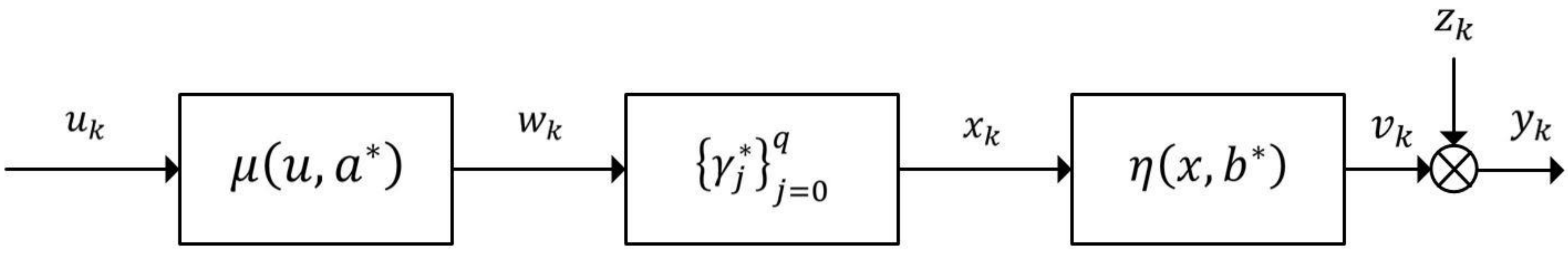
Statement of the problem

SISO Hammerstein-Wiener (N-L-N) system



$$y_k = \eta\left(\sum_{j=0}^{q} \gamma_j^* \mu(u_{k-j}, a^*), b^*\right) + z_k$$

$$y_k = v_k + z_k,$$

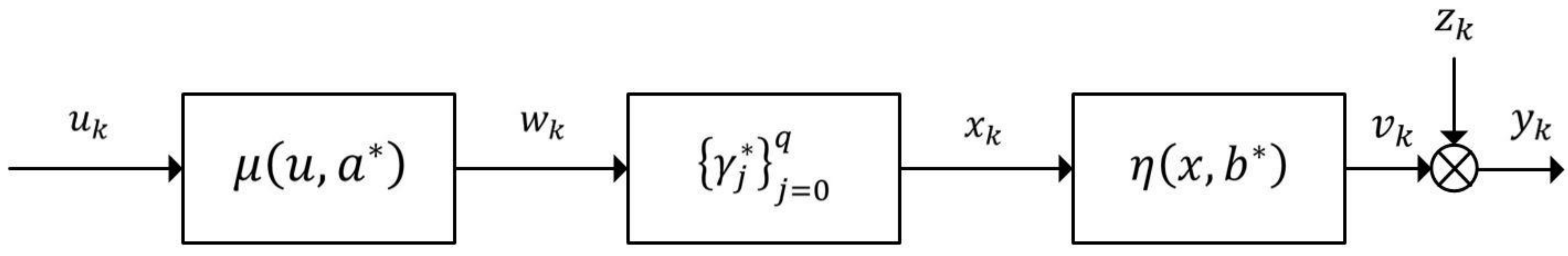
$$v_k = \eta(x_k),$$

$$x_k = \sum_{j=0}^{q} \gamma_j^* w_{k-j},$$

$$w_k = \mu(u_k)$$

Statement of the problem

SISO Hammerstein-Wiener (N-L-N) system



$$\mu(u) = \mu(u, a^*) = a_1^* f_1(u) + a_2^* f_2(u) + \cdots + a_m^* f_m(u),$$

$$a^* = (a_1^*, a_2^*, \ldots, a_m^*)^T, a^* \in \mathbb{R}^m,$$

$$\eta(x) = \eta(x, b^*) = b_1^* g_1(x) + b_2^* g_2(x) + \cdots + b_n^* g_n(x),$$

$$b^* = (b_1^*, b_2^*, \ldots, b_n^*)^T, b^* \in \mathbb{R}^n$$

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General assumptions

Regarding the system:

- Linear dynamic block is a FIR filter of known order q.
- Static nonlinear characteristics are Lipschitz functions described by the combination of *a priori* known base functions f and g. Characteristic $\eta(\cdot)$ is strictly monotonous.

Concerning input and noise signals:

- $u_k^{(1)}$ is an *i.i.d.* random process with Lipschitz probability density function f, and $f(0) > 0$.
- $u_k^{(2)}$ is an *i.i.d.* random and binary process.
- Input and noise signals are mutually independent and have finite variances. Expected value of noise signal equals zero.

Purpose of identification

$$Q(\gamma, a, b) = E(y_k - \bar{y}_k)^2 \rightarrow \min_{\gamma, a, b} ,$$

$$y_k = y_k(\gamma^*, a^*, b^*)$$

$$\bar{y}_k = \bar{y}_k(\gamma, a, b)$$

The algorithm

Stages

- Recovery of the finite impulse response γ^*
- Estimation of second nonlinear characteristic (parameters b^*)
- Creation of additional signal r_k with the same expected value as inaccessible signal x_k
- Identification of first nonlinear characteristic (parameters a^*)

Methods

- Kernel-censored least squares estimate
- Kernel-based method with the help of binary sequences
- Usage of reversed function $\eta^{-1}(\cdot)$ and probability density function $f(z)$
- Least squares method as in Hammerstein system identification

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Identification of linear block

$$\tilde{\gamma} = \left(\sum_{k=1}^{N} \phi_k \phi_k^T K\left(\frac{\Delta_k}{h_1}\right)\right)^{-1} \cdot \left(\sum_{k=1}^{N} \phi_k y_k K\left(\frac{\Delta_k}{h_1}\right)\right)$$

$$\phi_k = \left(u_k, u_{k-1}, \ldots, u_{k-q}\right)^T$$

$$\Delta_k = \|\phi_k\|_\infty = \max_{j=0,1,\ldots,q} |u_{k-j}|$$

$$K\left(\frac{\Delta_k}{h_1}\right) = \begin{cases} 1, as\ |\Delta_k| \le h_1, \\ 0, elsewhere. \end{cases}$$

Identification of linear block

Theorem 1:

Let given assumptions be in force. Then, for the Hammerstein-Wiener system and $\mathrm{h} \sim N^{-\alpha}$, where $\alpha \in \left(0, \frac{1}{d}\right)$ and $d = q + 3$, it holds that

$$\hat{\gamma}_j \to \gamma_j^*, j = 0, 1, \ldots, q$$

in probability as $N \to \infty$, provided that $c = \mu'(u_0)\eta'(x_0) \neq 0$.

Identification of linear block

Proof (sketch):
Let's take Taylor series expansion and apply it to input and output nonlinearities around points u_0 and x_0:

$$\mu(u_k) = \mu(u_0) + c_1(u_k - u_0) + \rho(u_k),$$
$$\eta(x_k) = \eta(x_0) + c_2(x_k - x_0) + \xi(x_k),$$

where $c_1 = \mu'(u_0),\ c_2 = \eta'(x_0)$.
Observations:
$\rho(u_k), \xi(x_k)$ are second order Taylor's rests and:

$$|\rho(u_k)| = o(h),$$
$$|\xi(x_k)| = o(h).$$

Estimation of second nonlinear characteristic

Binary representation of number $i-1, i=1,2,\ldots,N_0, N_0=2^{q+1}$

$$
\begin{aligned}
d_1 &= (0,0,\ldots,0,0,0)^T,\\
d_2 &= (0,0,\ldots,0,0,1)^T,\\
d_3 &= (0,0,\ldots,0,1,0)^T,\\
&\vdots\\
d_{N_0} &= (1,1,\ldots,1,1,1)^T.
\end{aligned}
$$

$$x_{[i]} = d_i^T \gamma^*$$

$$x_{[1]}, x_{[2]}, \ldots, x_{[i]}, \ldots, x_{[N_0]}$$

Estimation of second nonlinear characteristic

$$\hat{\eta}\left(x_{[i]}\right) = \frac{\sum_{k=1}^{N} y_k \delta(\phi_k, d_i)}{\sum_{k=1}^{N} \delta(\phi_k, d_i)}$$

$$\delta(\phi_k, d_i) = \begin{cases} 1, & as\ \phi_k = d_i \\ 0, & elsewhere \end{cases}$$

$$P\{\delta(\phi_k, d_i) = 1\} = \frac{1}{N_0} = \frac{1}{2^{q+1}}$$

Estimation of second nonlinear characteristic

$$y_{[1]}, y_{[2]}, \dots, y_{[L]},\ L\ - \text{random}$$

$$S\left(x_{[i]}\right) \triangleq \{y_k \mid \phi_k == d_i\}$$

$$\hat{\eta}\left(x_{[i]}\right) = Avg\left(S\left(x_{[i]}\right)\right) = \begin{cases} \frac{1}{L}\sum_{l=1}^{L} y_{[l]}, \text{if } L > 0 \\ 0, \text{otherwise} \end{cases}$$

Estimation of second nonlinear characteristic

Theorem 2:

Under give assumptions, it holds that

$$E\left[\eta\left(x_{[i]}\right) - \hat{\eta}\left(x_{[i]}\right)\right]^2 \to 0, \text{as } N \to \infty,$$

in each estimation point $x_{[i]} = d_i^T \gamma^*$,
$i = 1,2, \ldots, N_0$, such that $x_{[i]} \in cont\left(\eta(\,), g(\,)\right)$,
where $cont\left(\eta(\,), g(\,)\right)$ is a set of all point of continuity of $\eta(\,)$ and $g(\,)$.

Estimation of second nonlinear characteristic

Proof (sketch):

Using Wald identity and second form of the estimation function:

$$E\,\hat{\eta}(x_{[i]}) = \eta(x_{[i]}) + E\left(\frac{\sum_{k=1}^{N}\left(z_k \cdot \delta(\phi_k, d_i)\right)}{\sum_{k=1}^{N} \delta(\phi_k, d_i)}\right) = \eta(x_{[i]}) + \frac{EL \cdot Ez_1}{EL}$$

$$EL = P(\phi_k = d_i) \cdot N = \frac{1}{N_0} \cdot N = \frac{N}{2^{q+1}}$$

and benefitting from conditional variances:

$$var\,\hat{\eta}(x_{[i]}) = \sum_{k=1}^{N} P(L = k) \cdot \frac{\sigma_z^2}{k} = \frac{N_0 \sigma_z^2}{N} = c \cdot \frac{1}{N} \sim N^{-1}$$

Estimation of second nonlinear characteristic

$$\left\{\left(x_{[i]}, \hat{\eta}\left(x_{[i]}\right)\right)\right\}_{i=1}^{N_0}$$

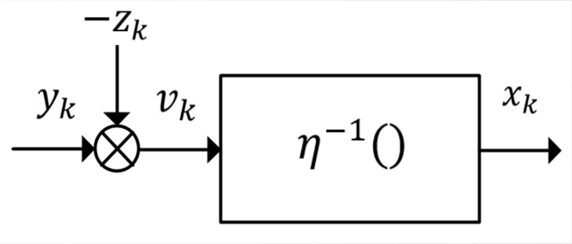
$$\hat{b} = (\Psi^T \Psi)^{-1} \cdot \Psi^T \zeta$$

$$\Psi = \left(\psi_1^T, \psi_2^T, \dots, \psi_{N_0}^T\right)^T$$

$$\psi_i = \left(g_1\left(x_{[i]}\right), g_2\left(x_{[i]}\right), \dots, g_n\left(x_{[i]}\right)\right)^T$$

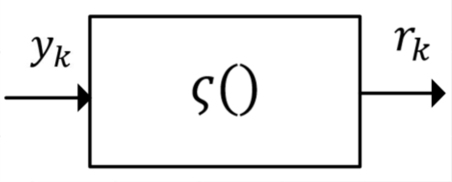
$$\zeta = \left(\hat{\eta}\left(x_{[1]}\right), \hat{\eta}\left(x_{[2]}\right), \dots, \hat{\eta}\left(x_{[N_0]}\right)\right)^T$$

Output signal filtration



$$y_k = \eta(x_k) + z_k$$

$$x_k = \eta^{-1}(y_k - z_k)$$



$$r_k = \varsigma(y_k)$$

$$\varsigma(y) = E\{x_k | y_k = y\} = \int_{-\infty}^{\infty} \eta^{-1}(y - z) f(z) dz$$

$$Er_k = E\varsigma(y_k) = E \int_{-\infty}^{\infty} \eta^{-1}(y_k - z) f(z) dz =$$

$$= E \int_{-\infty}^{\infty} \eta^{-1}(\eta(x_k)) f(z) dz = Ex_k \cdot \int_{-\infty}^{\infty} f(z) dz = Ex_k$$

Identification of first nonlinear characteristic

$$x_k = \phi_k^T \cdot \theta^*$$

$$\lambda_k = \Big(f_1(u_k), \dots, f_m(u_k), f_1(u_{k-1}), \dots,$$
$$f_m(u_{k-1}), \dots, f_1\big(u_{k-q}\big), \dots, f_m\big(u_{k-q}\big)\Big)^T$$

$$\theta^* = \left(\gamma_0^* a_1^*, \dots, \gamma_0^* a_m^*, \gamma_1^* a_1^*, \dots, \gamma_1^* a_m^*, \dots, \gamma_q^* a_1^*, \dots, \gamma_q^* a_m^*\right)^T$$

$$\hat{\theta} = (\Lambda_N^T \Lambda_N)^{-1} \cdot \Lambda_N^T R_N$$

Numerical example

$$N = 10^5$$

$$u_k^{(1)} \sim U[-1,1]$$

$$u_k^{(2)} \sim B(0,1)$$

$$z_k \sim U[-0.5,0.5]$$

$$\gamma^* = [0.6\ 0.3\ 0.1]^T$$

$$a^* = [0.8\ 0.2]^T$$

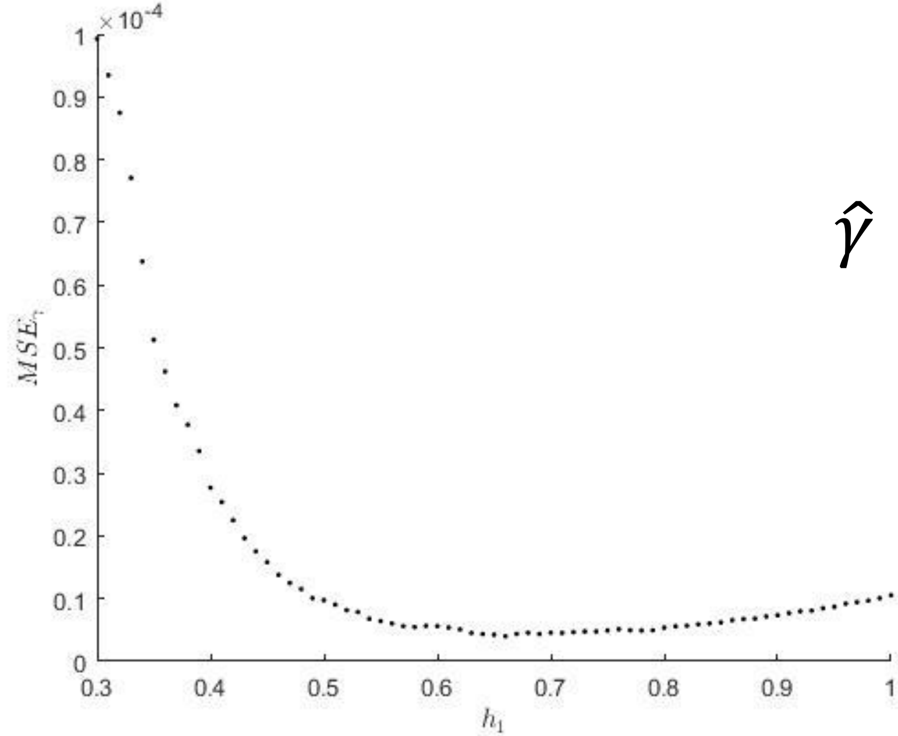
$$b^* = [0.7\ 0.4]^T$$

$$\mu(u) = a_1^* u + a_2^* u^2$$

$$\eta(x) = b_1^* x + b_2^* x^2$$

Estimation of parameters α^*

$$E_\gamma = \frac{1}{q+1}\sum_{i=0}^{q}(\gamma_i^* - \hat{\gamma}_i)^2 = 1.38 \cdot 10^{-4}$$

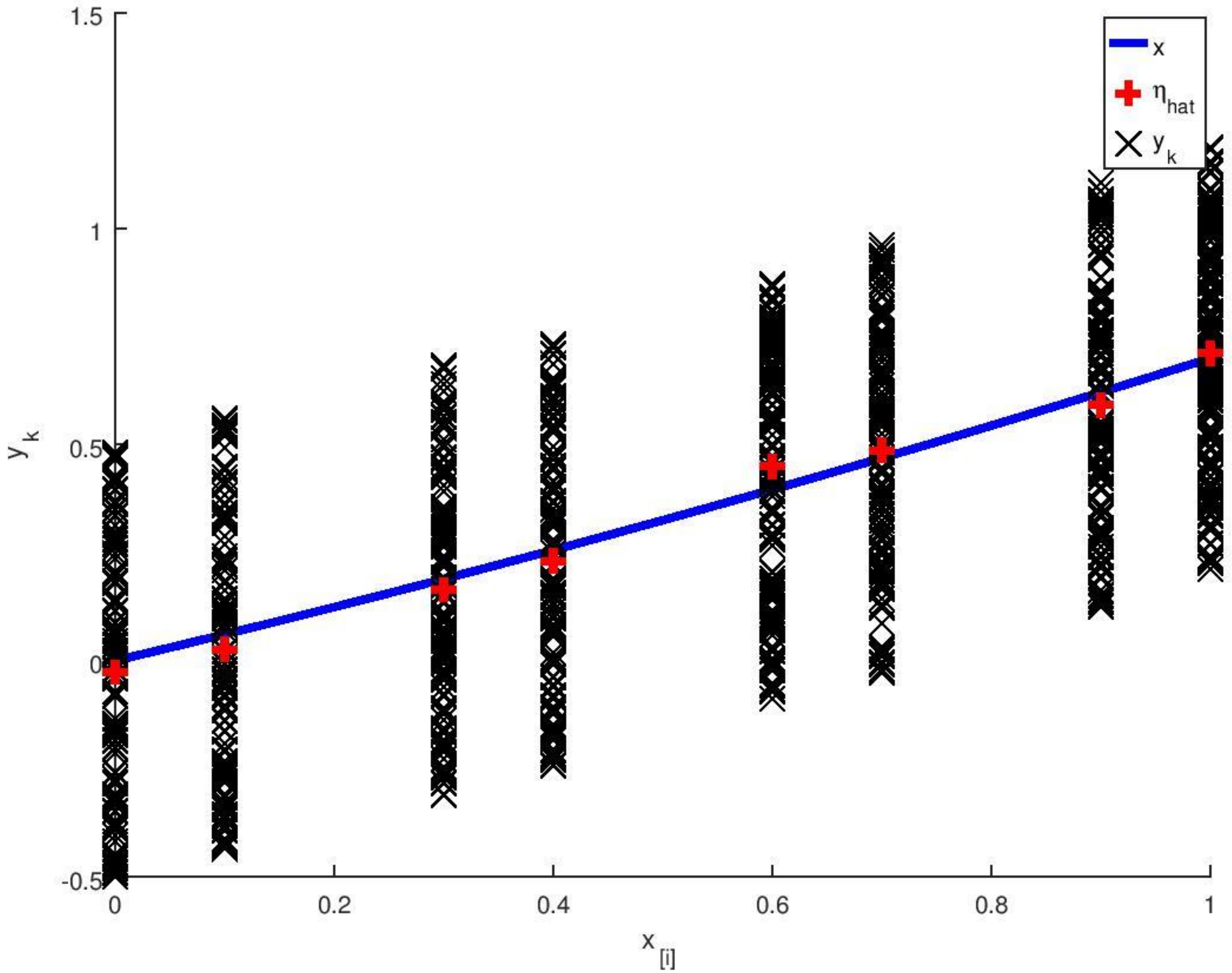


$$\gamma^* = [0.6\ 0.3\ 0.1]^T$$

$$\hat{\gamma} = [0.613\ 0.302\ 0.085]^T$$

$$h = 0.66$$

Estimation of parameters b^*



Estimation of parameters $\boldsymbol{b}^*$

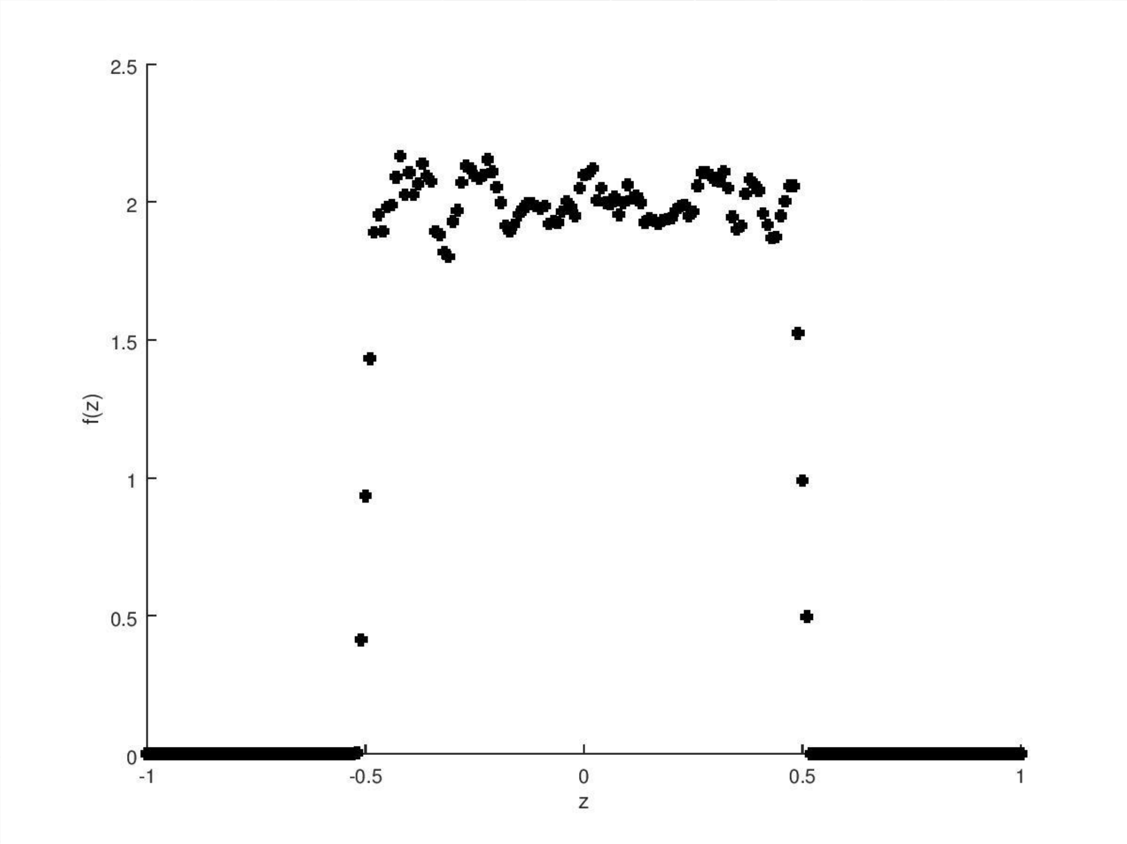
$$b^* = [0.7\ 0.4]^T$$

$$\hat{b} = [0.699\ 0.401]^T$$

$$E_b = \frac{1}{2}\sum_{i=1}^{2}\left(b_i^* - \hat{b}_i\right)^2 = 1.1 \cdot 10^{-6}$$

Kernel estimation of $f(z)$

$$f(z) = \frac{1}{Nh}\sum_{k=1}^{N} K\left(\frac{z_k - z}{h}\right)$$



Output signal filtration

1) Fast Fourier transform
2) Numerical integration with Riemann sum
3) Monte Carlo method

$$\hat{\zeta}(y) = \frac{1}{n}\sum_{i=1}^{n} \eta^{-1}(y - z_i)$$

Applications

- Modelling both actuator and sensor nonlinearities
- Modeling physical, chemical, biological processes:
  - polymerase reactors
  - ionospheric processes
  - pH processes
  - magnetospheric dynamics
- Modelling electrically simulated muscles
- Modelling power amplifiers, electrical drives
- Modelling magneto-rheological dampers

Conclusions

- Combination of both parametric and nonparametric approaches
- The system is identifiable and the solution is unique for the impulse response fulfilling the given assumptions and for output nonlinearity satisfying Haar condition
- Division of the identification into stages significantly reduces dimensionality of the system identification
- Effectiveness is strictly related to the length of the impulse response
- Recommended for dynamic filters with short memory and more elaborated nonlinearities
- The algorithm for H-W system identification can adapt itself both to Hammerstein and Wiener systems separately without any additional knowledge about the examined system

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Thank you for your attention

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The work was supported by the National Science Centre, Poland, Grant No. 2016/21/B/ST7/02284