



$$\bar{e}_i \circ \bar{e}_j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$\bar{v} \circ \bar{e}_1 = v_1 \quad \checkmark$$

$$\bar{v} \circ \bar{e}_2 = v_2$$

$$\bar{v} \circ \bar{e}_3 = v_3$$

$$\checkmark \quad \bar{v} = v_1 \bar{e}_1 + v_2 \bar{e}_2 + v_3 \bar{e}_3 = \\ = \langle \bar{v}, \bar{e}_1 \rangle \bar{e}_1 + \langle \bar{v}, \bar{e}_2 \rangle \bar{e}_2 + \langle \bar{v}, \bar{e}_3 \rangle \bar{e}_3$$

$$\bar{v} \circ \bar{v} = (v_1 \bar{e}_1 + v_2 \bar{e}_2 + v_3 \bar{e}_3) \circ (v_1 \bar{e}_1 + v_2 \bar{e}_2 + v_3 \bar{e}_3) =$$

$$\checkmark \quad = v_1^2 + v_2^2 + v_3^2 = \|v\|^2 \leq R$$

$$f(x) = \sum_{i=1}^{\infty} a_i \varphi_i(x)$$

gdzie

$$a_i = f(x) \circ \varphi_i(x)$$

DANE:

$$\{\varphi_i(x)\}_{i=1}^{\infty}$$

$$\varphi_i \circ \varphi_j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$f(x) \circ f(x) = \int f^2 dx = \|f(x)\|^2 =$$

$$= (a_1 \varphi_1(x) + a_2 \varphi_2(x) + a_3 \varphi_3(x) + \dots) \circ (a_1 \varphi_1(x) + a_2 \varphi_2(x) + \dots)$$

$$= a_1^2 + a_2^2 + a_3^2 + \dots = \sum_{i=1}^{\infty} a_i^2 \leq R \quad \text{RÓWNOŚĆ PARSEVALA}$$

$$\|f\| \leq R < \infty$$

